NUMERICAL ANALYSIS OF THERMOMECHANICAL BEHAVIOR OF A MULIMATERIAL UNDER THERMAL CYCLING CONDITIONS

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ABSTRACT

The increasing need of structures having multiple functions orientates designers to combine materials in order to obtain, according to coupling scales, multi-materials structures.

The lifetime of these structures represents the essential decisive element for study offices and manufacturers. The results of this work should add to the set of functional charges and constraints of resistance, the improvement of the lifetime as an objective to optimize a multi-material.

In this study, we propose a numerical analysis by the finite elements method of the thermo mechanical behavior of these materials and their damage under thermal cyclic solicitations. The sample is a two-dimensional plate constituted of two different isotropic layers (steel, aluminum) submitted to variable thermal conditions (heat flux condition on one side and convection exchange condition on the opposite side). The sample is supposed to be fixed in one direction and free in the other. The damage model is based on the works of Lemaître and Chaboche [8].

Numerical results are presented for different forms of heat flux cycling (triangular, square and sinusoidal excitations) with a comparison of the multi-material damage for each excitation.

The study is concluded by an empirical optimizing of the thickness of materials according to the total lifetime caused by thermo-mechanical effect.

INTRODUCTION

Every system or mechanical component is likely to be subject to a thermo-mechanical coupling (steel constructions, mechanical machines, motors, turbines, etc....). However, this heating effect varies radically between fields. On the one hand, it is negligible in the steel constructions; on the other hand, it is very significant on the cylinder of an engine. This is due to the fact that the abrupt variation and the periodicity of the temperature influence the fatigue of constituent materials. That leads to the destruction of materials and of the engine as a whole in consequence.

Few works have been devoted to a local study of damage in the composites in transient state under variable thermal conditions in space as well as at one time. Indeed, most of studies have been realized in steady thermal state [1] or in unsteady state by supposing a negligible thermal resistance to each instant [2]. General manner, these approaches underestimate the thermo-elasto-plastic behavior of these materials and as a result their behavior in fatigue [3].

In this article, we're carrying out an analysis of the damage in two layer isotropic materials (steel, Aluminum) in the transient state. The body is submitted to a thermal cycling heat flux on one face and an exchange by convection with ambient face in the opposite.

This work is organized as follows: A description of the studied sample is detailed in section I, the physical formulation of the thermal problem, coupling and damage is shown in the second part. In part III, we carry out a numerical modeling of the problem using Matlab and we report the results obtained and analyses resulting from each case apart. Optimizations which are drawn from the results obtained by the numerical model are presented in section IV, the conclusion of this paper is given in section V.

I- Presentation of the problem

The studied body consists of two layers of different materials, the sub-base A1 is a layer made of steel and the other A2 is made of aluminum (see figure 1). The physical specifications of these two materials are indicated in table 1.

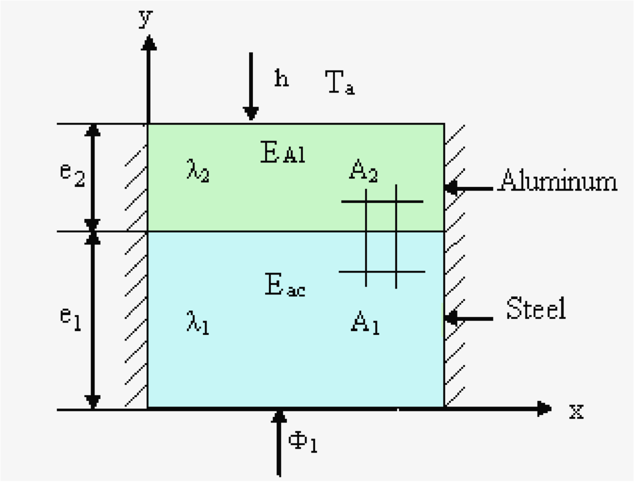

Figure 1: The Studied Sample

We apply to the sub-layer of the sample, a sinusoidal heat flux φ1 for maximum heat flux equal to 8000 W/m, and to the superficial layer, a convection exchange whose coefficient h=6 W/m ° K. The initial temperature (ambient) is equal to 300 ° K. We consider that both left and right sides are isolated.

	steel	Aluminum
ρ: density	7854 kg/m3	2770 kg/m3
c: heat capacity	434 J/Kg.°K	875 J/Kg.°K
λ: thermal conductivity	60.5 W/m°K	177 W/m°K
L: length	0.7 m	0.7m
l: Width	l1 = 0.04 m	l2 = 0.03 m

Table 1: Physical Specifications of Materials [6]

II- MATHEMATICAL FORMULATION

Our approach consists of considering the thermal problem apart, to observe the thermal behavior of the body and the variation of the temperature at each point of it with respect to the excitation applied, then we consider the mechanical problem to observe the dilation and the deformations which will seem to vary according to the temperature applied at each point, and also to notice the constraints which are exerted as a result of these excitations.

1- Thermal Formulation

It is to be noted here that in our study, we're interested in the transient state. In the inner parts of the two bodies and at each point, the essential mathematical equation which is applied to the system is as follows:

$$(\rho c_p)_i \cdot \frac{\partial T_i}{\partial t} = div(\lambda_i \ \overrightarrow{grad}\ T_i) \qquad i = 1,2 \quad (1)$$

With i= 1 (Steel)
i= 2 (Aluminum)

The boundary conditions or the equations of transfer of heat to the borders are written in the following way:

$$-\lambda_1 \frac{\partial T}{\partial y} = \phi_1 \text{ (Sub-base)}$$

$$-\lambda_2 \frac{\partial T}{\partial y} = h(T_p - T_0) \text{ (Roadbase)}$$

$$-\lambda_i \left.\frac{\partial T}{\partial x}\right|_{x=0} = -\lambda_i \left.\frac{\partial T}{\partial x}\right|_{x=L} = 0 \qquad i = 1,2 \text{ (Right and left side)}$$

Between the two materials, the equation of exchange of the heat flux is written as follows:

$$-\lambda_1 \frac{\partial T}{\partial y} = -\lambda_2 \frac{\partial T}{\partial y}$$

2 – Mechanical Formulation

The formulation for Lamé Law can be written as follows [5]:

$$\overline{\overline{\sigma}} = 2\mu\overline{\overline{\varepsilon}} + \lambda tr(\overline{\overline{\varepsilon}}).I \quad (2)$$

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix} = 2\mu \begin{bmatrix} \varepsilon_x & \gamma_{xy} \\ \gamma_{xy} & \varepsilon_y \end{bmatrix} + \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} (\varepsilon_x + \varepsilon_y) \ (3)$$

With $\mu_i = \dfrac{E_i}{2(1+\upsilon_i)}$, $\lambda_i = \dfrac{\upsilon_i E_i}{(1-2\upsilon_i)(1+\upsilon_i)}$ i=1, 2

3- Thermo mechanical Coupling

The thermo-mechanical coupling is modeled by these equations [6]:

$$\sigma_{ij} = E_{ij}(\varepsilon_{ij} - \alpha_{ij}(T(y,t) - T_0)), \ (4)$$

In matrix form for an isotropic material in 2D, $\alpha_{xy} = 0$

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{xy} \end{bmatrix}_i = \frac{E_i}{(1+\upsilon)(1-2\upsilon)} \begin{bmatrix} 1-\upsilon & \upsilon & 0 \\ \upsilon & 1-\upsilon & 0 \\ 0 & 0 & \frac{1-2\upsilon}{2} \end{bmatrix}_i \begin{bmatrix} \varepsilon_{xx} - \alpha(T(t) - T_0) \\ \varepsilon_{yy} - \alpha(T(t) - T_0) \\ \varepsilon_{xy} \end{bmatrix}_i \ (5)$$

4- Damage model

For a one-dimensional model and for elasticity coupled to damage the energy function $W^e = \frac{1}{2}E(1-D)(\varepsilon^e)^2$ (6) can be written down in an intuitive way since damage decreases the elasticity of the material by a ratio (1-D). Following Kachanov (1958) and Rabotnov (1963) [4], to whom the idea is due, we can write $\sigma = \dfrac{\partial W^e}{\partial \varepsilon^e} = (1-D)E\varepsilon^e$ (7)

Hence $\varepsilon^e = \dfrac{\sigma}{(1-D)E} = \dfrac{\sigma}{\overline{E}} = \dfrac{\overline{\sigma}}{E}$ (8)

Where $\overline{\sigma} = \dfrac{\sigma}{(1-D)}$ is the effective stress and $\overline{E}$ is the effective elasticity modulus resulting from damage.

For linear isotropic elasticity, we would obtain

$$Y = \frac{\sigma_y^2}{2E(1-D)^2} R_v \ (9)$$

Where $\sigma_y = \left(\frac{3}{2}\sigma_{ij}^d \sigma_{ij}^d\right)^{\frac{1}{2}}$ (10), $\sigma_{ij}^d = \sigma_{ij} - \sigma_m \delta_{ij}$ (11) and $\sigma_m = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ (12) are respectively the equivalent stress, the deviatoric stress, and the mean stress, and we have set:

$$R_v = \frac{2}{3}(1+v) + 3(1-2v)\left(\frac{\sigma_m}{\sigma_y}\right)^2 \quad (13)$$

E and v are respectively Young's modulus and Poisson's ratio for the non-damaged material. The dimensionless quantity R_v can be viewed as defining a damage criterion by (Lemaitre & Chaboche [8])

$$\sigma_y \sqrt{R_v} - \sigma^* = 0 \quad (14)$$

For a one-dimensional model we can write:

$$\overline{E} = (1-D)E \Rightarrow D = 1 - \frac{\overline{E}}{E} \quad (15)$$

Both $\overline{E}$ and E can be carefully measured and experimental curves giving D as a function of the number of loading cycles in a fatigue-creep test at low frequency, figure 2. This exhibits a damage accumulation in fatigue process and a distinction needs to be made between low and high cycle fatigue. Typically, we would write $\frac{\delta D}{\delta N} = g(\sigma_m, \sigma_M, D)$ (16) where N is the number of cycles, σ_m is the mean stress and σ_M is the maximum stress. In the case of low-cycle fatigue, the damage increment in each cycle is typically related to the plastic strain accumulated during the cycle, while for high-cycle fatigue, the microcrack nucleation resulting in damage is thought to be related to stress concentrations at the tips of isolated slip bands. The choice of the function g above, requires some ingenuity. However, experimental results such as those in the figure 2 suggest the possibility of using a function of damage having the following variation $\dot{D} = \frac{\partial D}{\partial N} = \left(\frac{Y}{S_0}\right)^{s_0} \frac{1}{1-D}$ (17)

Where only two material parameters, S_0 and s_0, are present [4].

$$\frac{\partial D}{\partial N} = \left(\frac{\sigma_x^2 R_v}{S_0 2E(1-D)^2}\right)^{s_0} \frac{1}{1-D} \quad (18)$$

In our case study, $\sigma_y = 0$, σ_x results from the submission of the sample to a cyclic thermal source and the fixation of the left and right sides.

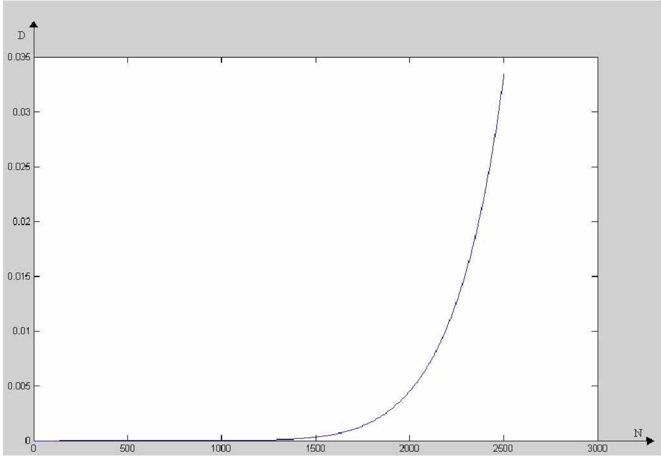

Figure 2: Evolution of the Damage in empirical approach (steel)

III- NUMERICAL ANALYSIS OF THE PROBLEM

1- Decomposition of the Body into Elements

Having a two-dimensional problem, we chose to break up the body into small pieces forming together nodes (i,j).

Our approach consists of transforming the two-dimensional problem (i, j) into a mathematical problem with only one dimension (k) with K = (j-1)*N+i.

There will then be a sum of MN nodes distributed on the studied body as indicates the following figure (finite differences method).

The decomposition of the body is done by considering Δx on the x axis, and Δy on the y axis, where Δy must be a common divider of e_1 and e_2 to guarantee obtaining nodes on the limit between the two plates. e_1 and e_2 being the respective thickness of the two plates, the common length of the two layers is L (see figure 1).

2- Numerical Analysis of the Thermal Phase

The goal of this phase is to make a Matlab program which calculates the temperature at each point of the body that is submitted to a heat flow. This calculation is done by the finite differences method.

The thermal equations quoted in II change numerically as follows:

* In each point located inside two materials

The general numerical equations take the following forms:

$$T_k^{n+1} = BT_{k+1}^n + BT_{k-1}^n + CT_{k+N}^n + CT_{k-N}^n + AT_k^n \quad (19)$$

This equation is represent by simple matrix form $(T^{n+1})=[P_2](T^n)$ (19).

* On the borders:

The simple form matrix is represent by:

$(T_k) = [P_1]\,(T_k) + (Q)$ (20)

Where $[P_1]$, $[P_2]$ (which are square matrices having MN order), and Q (which is a matrix column having a dimension MN), are defined by the exploitation of the equations above. T^n is the temperature vector which gives the value of the temperature in each point of the body studied at the instant n. The step of decomposition of time is dt.

This digital model can be implemented on the Matlab software easily. It is enough to define the matrices P_1, P_2 and Q and to use the recurrence of equations 26 and 27 in order to calculate the temperature in each point of the system at dt time intervals (in seconds), between the moment t = 0, and t = C s.

Results of the thermal analysis:

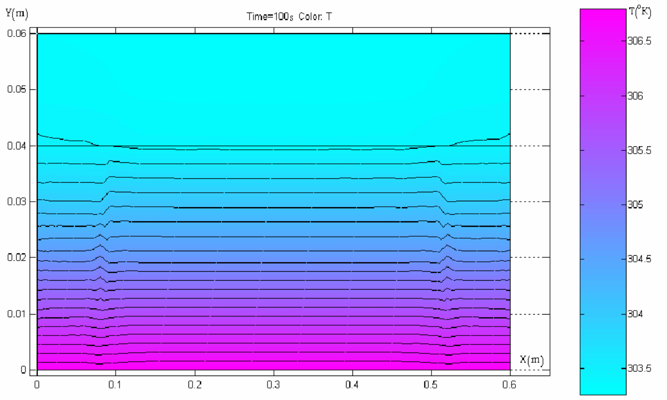

Figure 3: Fields of Temperatures at 360s on the Model in 2D

Each curve (with a different color) represents the variation of the temperature in a given point in the body (going from the point which is the nearest to the applied heat flux till we reach the furthest point).

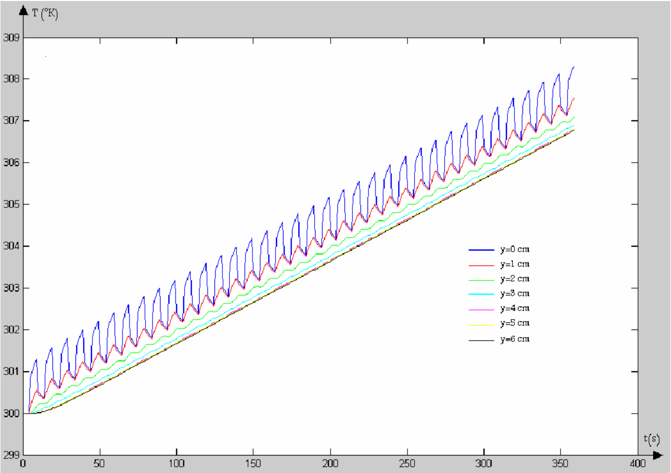

Figure 4: Evolution of temperature (Square Excitation case)

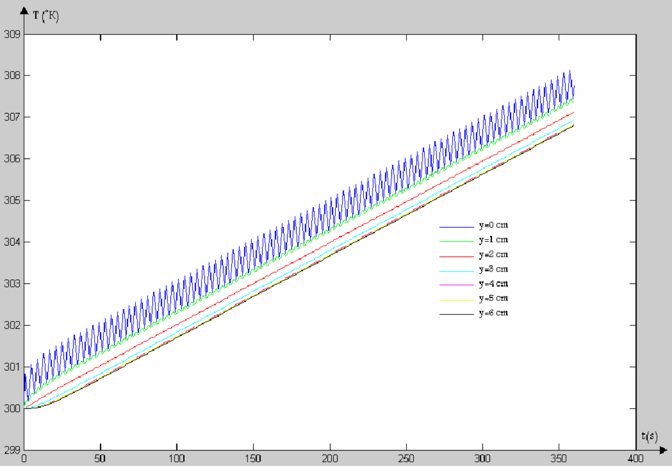

Figure 5: Evolution of temperature (Sinusoidal Excitation case)

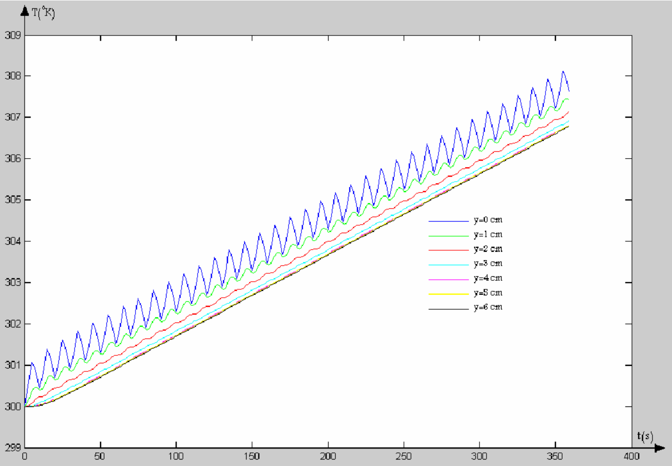

Figure 6: Evolution of temperature (Triangular Excitation case)

3- Thermo mechanical Analysis

The calculus can be easily done at the boundaries by using the equation 5, which is drawn from Lamé law, because we know the temperature at every position and moment. The latter is distributed in an isothermal way according to the longitudinal direction of the plate. It varies in a linear way in the vertical direction (see figure 3).

We also know deformations which are null in the x and y directions on the isolated and fixed embedded sides. On the other side $\sigma_{yy} = 0$ on the inferior and superior edges, this enables us to calculate the constraint in each point on the isolated side and consequently everywhere in the sample.

The maximum constraint resulting from the temperature takes place in the inferior face of steel (maximum temperature). We must also calculate the maximum constraint on the Aluminum which is perfectly stuck to the steel (see figures 4, 5, 6).

Results of the thermo mechanical analysis

The figures 4, 5, and 6 shows that the body is submitted to negative charges (fixed support and dilation of materials). Each curve (different colors on figures 7, 8, 9) represents the constraints applied to an horizontal set of the partitioned elements.

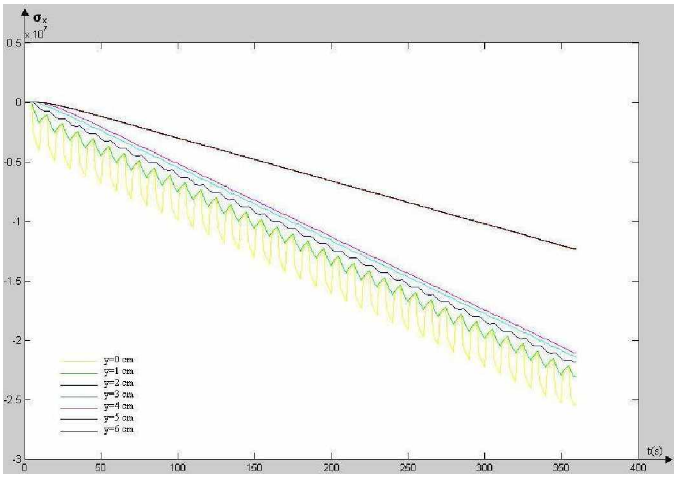

Figure 7: Thermo mechanical constraint σ_x, square excitation case

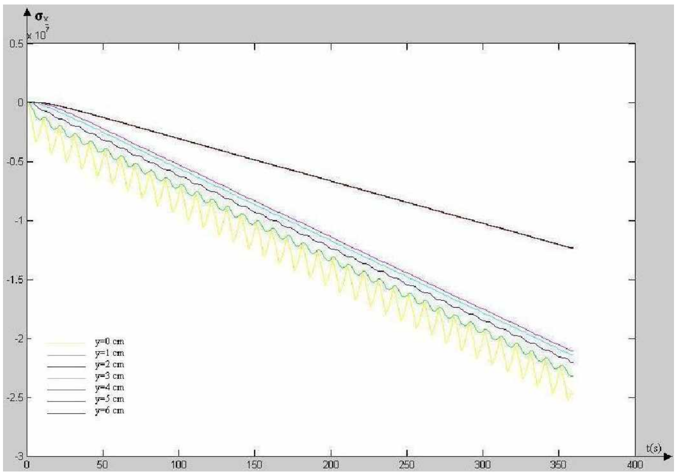

Figure 8: Thermo mechanical constraint σ_x, triangular excitation case

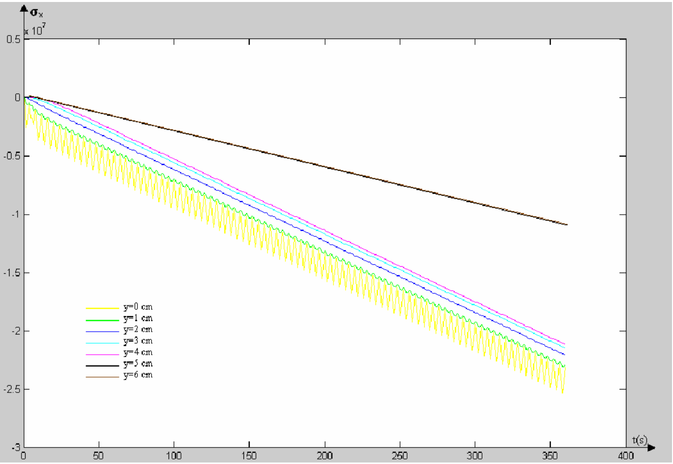

Figure 9: Thermo mechanical constraint σ_x, sinusoidal excitation case

The three preceding figures show that the values of the evolution of the constraints do not differ with those of the thermal excitation (sinusoidal, square or triangular) and this is due to the short period and amplitude of the applied excitation. That's why, and during the study of the damage exposed below, we only choose the sine wave excitation.

4- Numerical Analysis of the Damage

The damage study done in section II.4, see equation 18, gives the following numerical equation:

$$D_{i+1} = \Delta n K^{s_0} \frac{1}{(1-D_i)^{2s_0+1}} + D_i \quad \text{With } K = \frac{\sigma_x^2 R_v}{S_0 2E} \qquad (21)$$

s_0, S_0 are obtained from numerical calculation at a critical number of cycles equal to 10^7

Δn is the variation of number of cycles (Δn =1)

D_i is the damage value in the ith cycle

Results and Discussion

The implementation of the chosen law of damage, on the Matlab software enables us to calculate the damage on both studied materials. During our study, we have calculated the maximum constraints applied to steel and aluminum, these latter are applied on the sub-layers of both materials, and then we have also calculated the temperature and the damage.

We sought to carry out a comparison between the limits of damage by considering various percentages thicknesses of these two materials.

For e_{ac} = 0.01m, e_{Al} = 0.05 m, we present, in figures 10 and 11 the evolution of the damage with respect to the number of cycles respectively for steel and aluminum layers. It is worth noting here that the constraint obtained for the other percentages of thickness is identical to those shown below. That's why we will present for the other thicknesses only the figures of damage and the temperature of aluminum. It should also be noted that the damage of aluminum will take place before that of steel because of its low rupture stress.

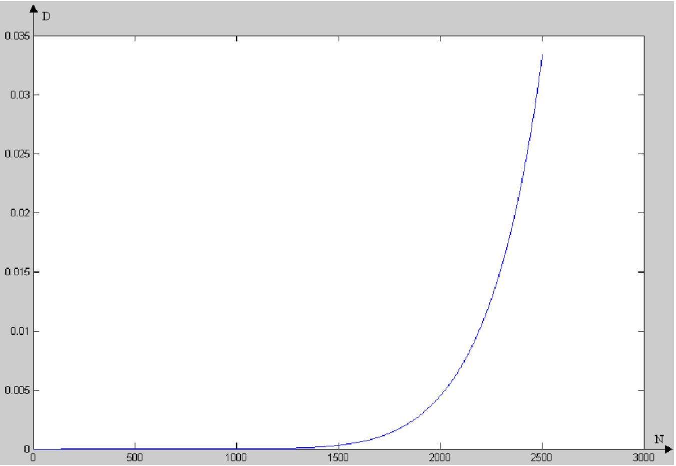

Figure 10: Damage of steel with respect to the number of cycles

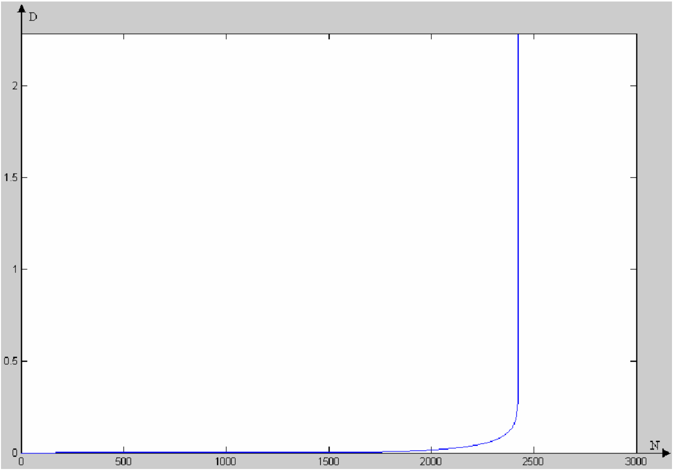

Figure 11: Damage of Aluminum with respect to the number of cycles

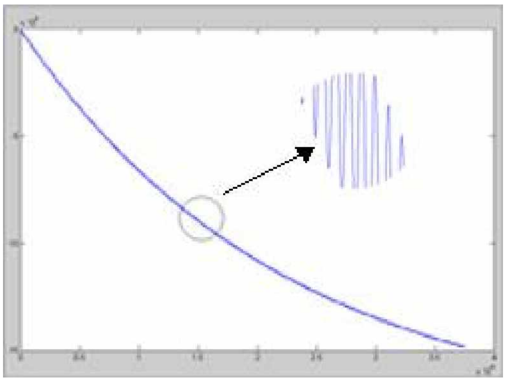

Figure 12: Steel Constraints

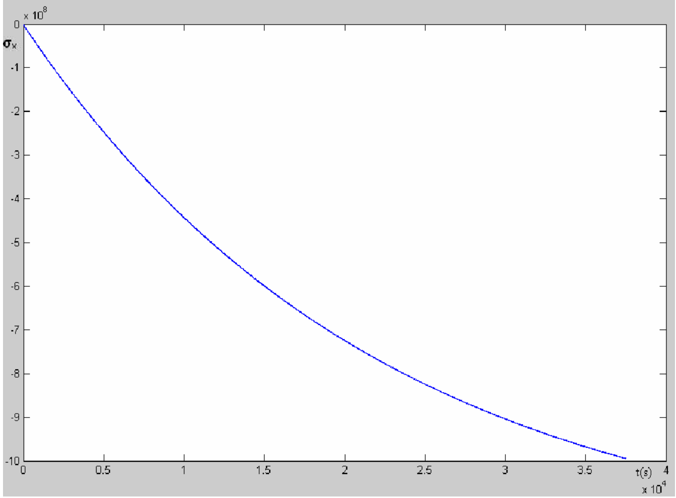

Figure 13: Aluminum Constraints

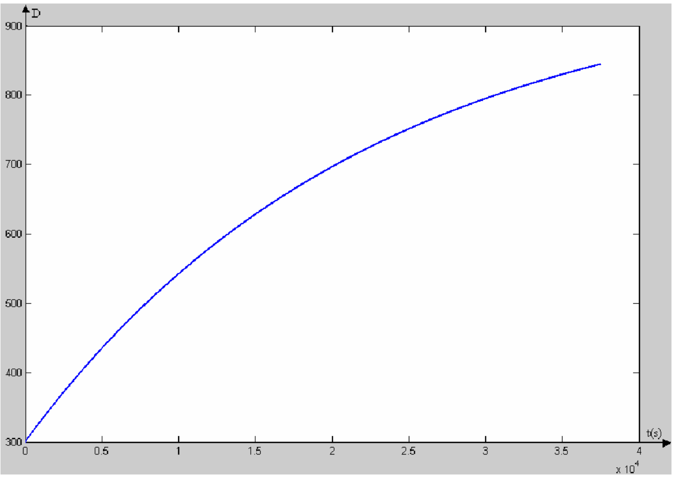

Figure 14: Maximum temperature on steel

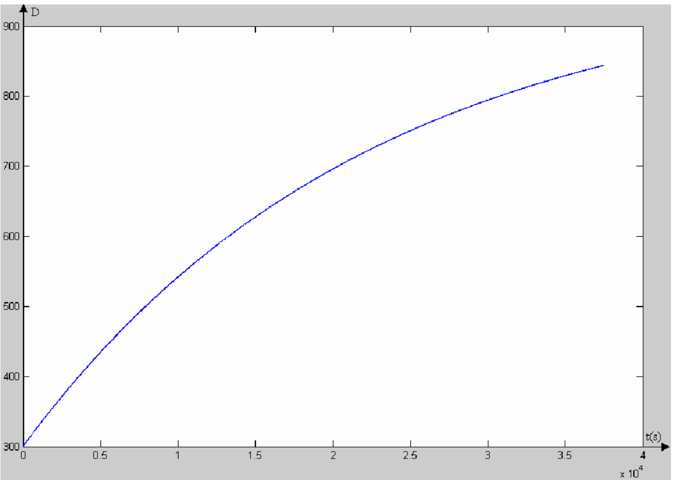

Figure 15: Maximum temperature applied on Aluminum

For e_{ac} = 0.02m, e_{al} = 0.04 m, we obtain:

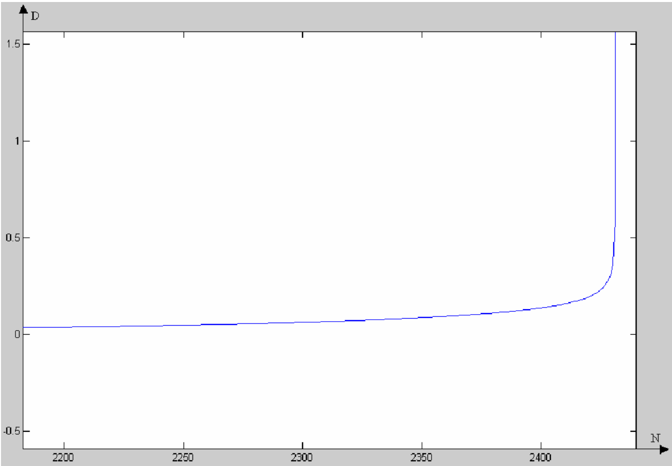

Figure 16: Damage of Aluminum

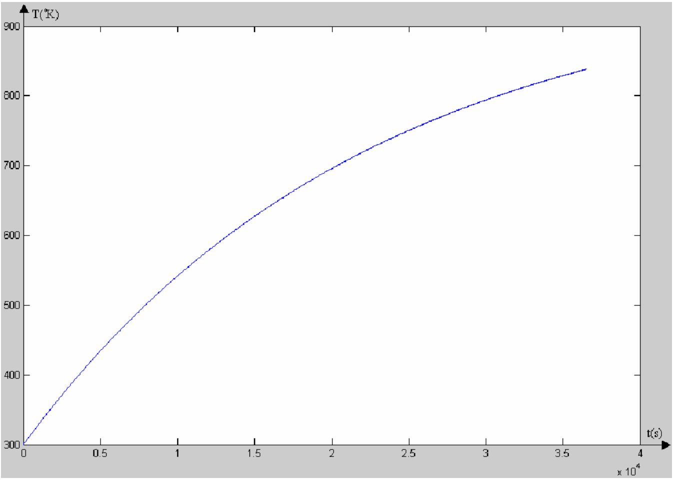

Figure 17: Temperature of Aluminum

For e_{ac} = 0.03m, e_{al} = 0.03 m, we got the following results:

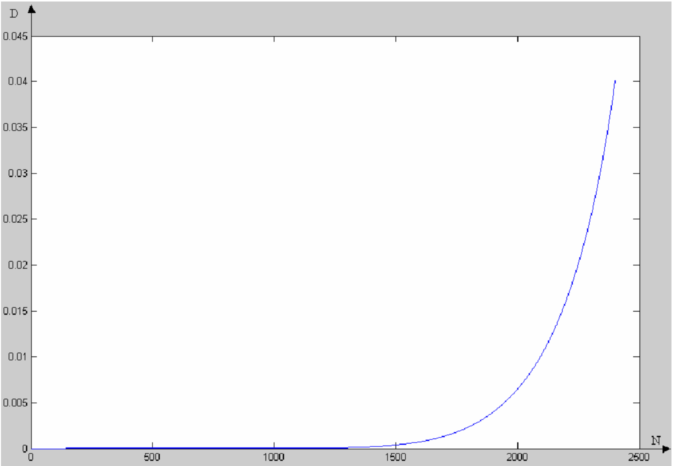

Figure 18: Damage of Aluminum

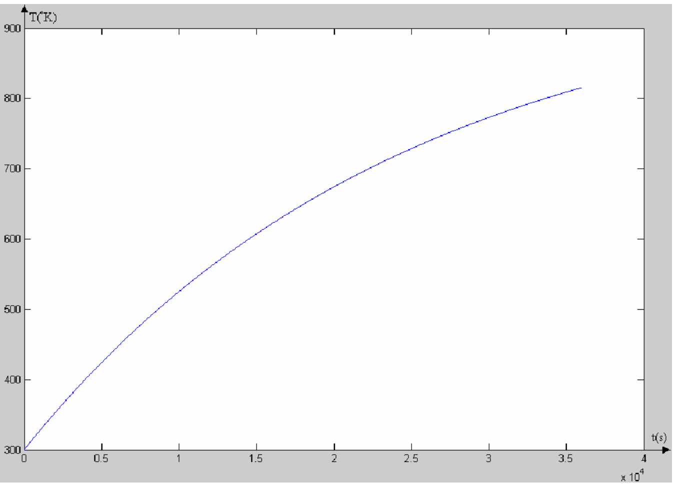

Figure 19: Temperature of Aluminum

For e_{ac} = 0.04m, e_{al} = 0.02 m, the following results are obtained

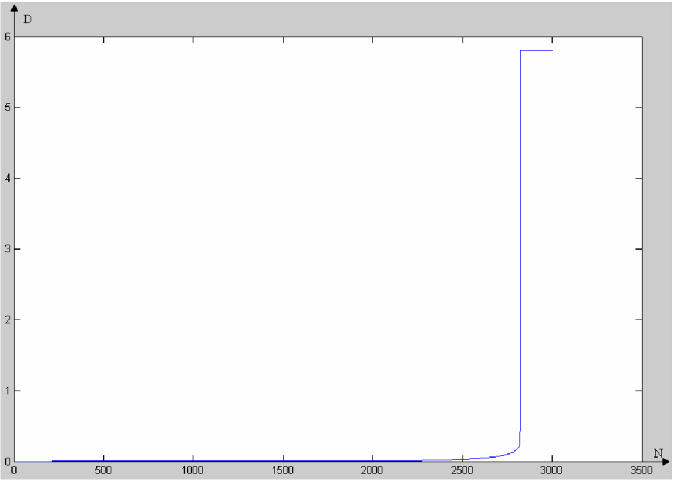

Figure 20: Damage of Aluminum

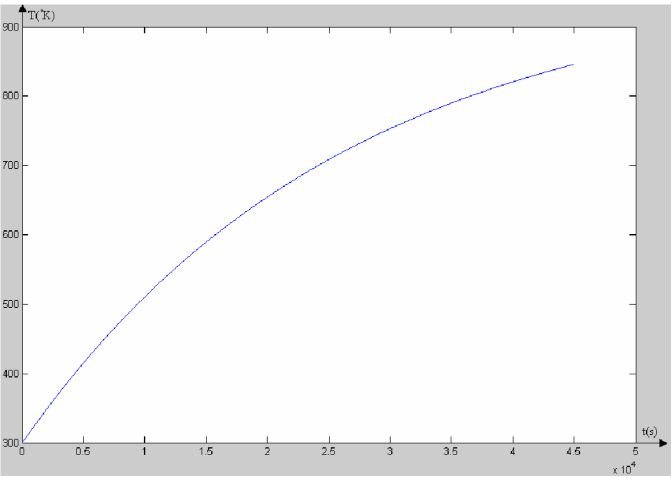

Figure 21: Temperature of Aluminum

And finally, for e_{ac} = 0.05m, e_{al} = 0.01 m, we obtain

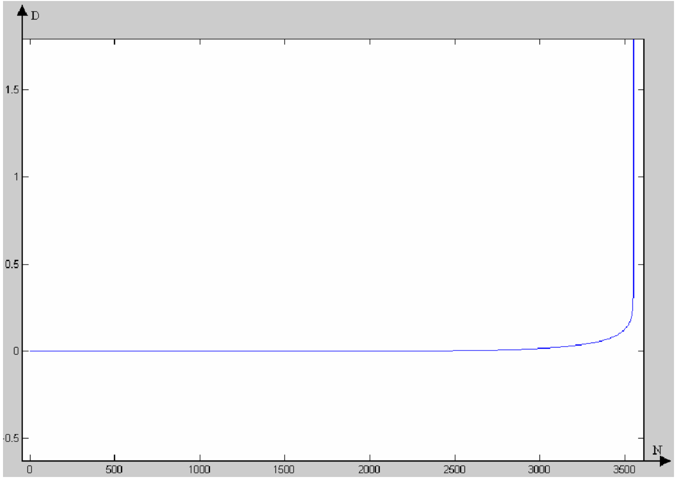

Figure 22: Damage of Aluminum

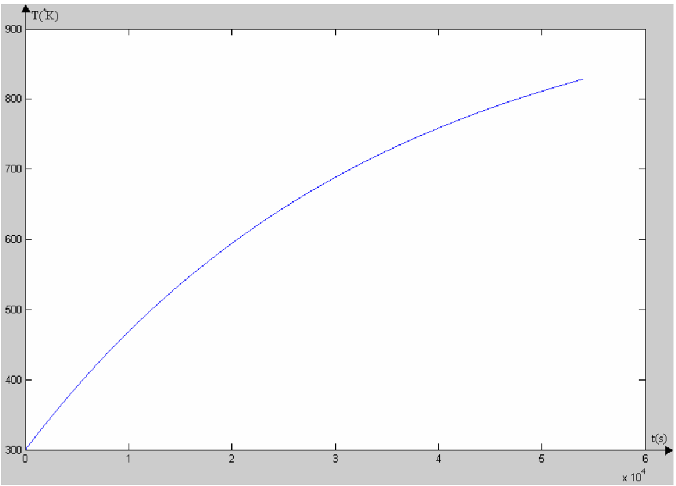

Figure 23: Temperature of Aluminum

IV EMPIRICAL OPTIMIZATION

To show the variation of the limit of damage with the variation of thicknesses of both materials, we can plot two curves which show the variation of the limit of damage fig. 24 (the maximum number of cycles causing the damage) and the temperature of damage, according to the thickness of the layer of steel. Figure 25.

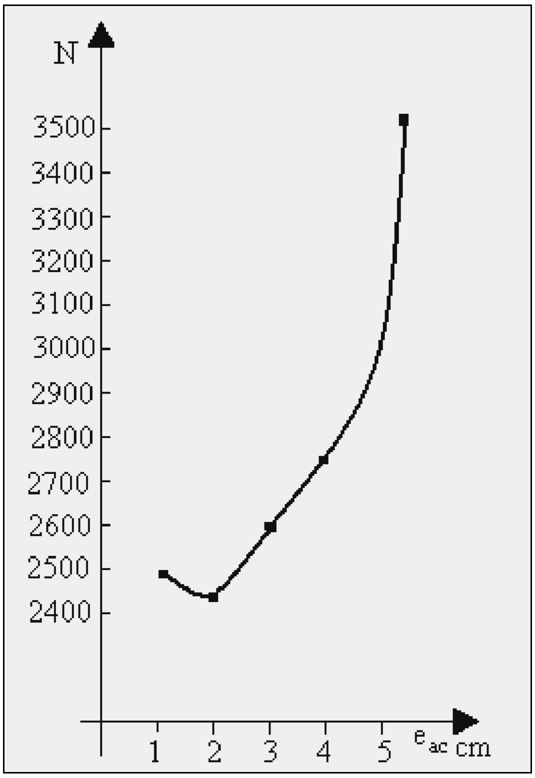

Figure 24: Limit number of cycles with respect to e_{ac}

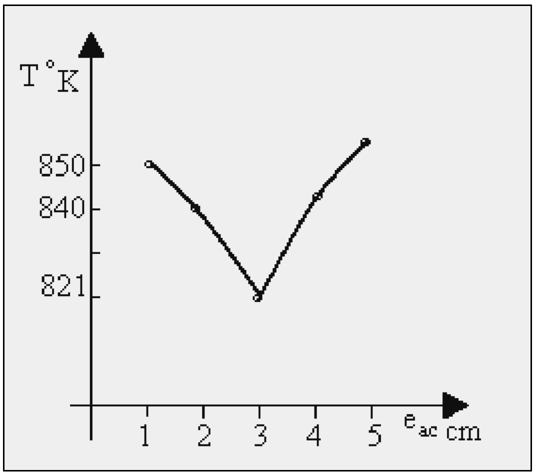

Figure 25: Temperature of damage of the AL with respect to e_{ac}

These figures show that for a thickness of steel equal to 3 cm, we have a minimum of the temperature in the body and a relatively large optimal value of the number of cycles that the body can support before undergoing a remarkable damage. However, for a thickness of steel equal to 5 cm, the body can be submitted to a great number of cycles without being damaged, but in this case, the temperature is higher and the body becomes heavier (almost totally made of steel). Thus, the optimal thickness of steel in our case is 3 cm.

V CONCLUSIONS and PERSPECTIVES

The manufacturers of multi materials having critical functions are interested in:

1- The lifetime that should be as long as possible.
2- The temperature undergone by the body which should not exceed precise limits.
3- The great resistance to the constraints.
4- The weak weight of the body.

For these reasons, and during our study, we sought to optimize the geometry of the multi materials in order to fulfill the requirements quoted above.

A significant result has been obtained during this study, it is that the damage is not influenced by the nature of the excitation applied (square, triangular or sinusoidal) as long as the period and the amplitude of the latter are not large.

Due to the existence of several parameters α, E, λ …. Which figure in the calculus, we can use the genetic algorithm in the optimization procedure.

In the same way, we envisage to make the study by the addition of a third layer in order to improve the optimization on the basis of the criteria quoted above.

The same study may be done in the future by extending the work done to three-dimensional. So 3D study may be of great value to researchers in the field of multi functional materials.

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