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NUMERICAL MODELLING OF FLOATING BREAKWATERS AND SHAPE OPTIMIZATION USING A NONLINEAR PROGRAMMING METHOD

ABSTRACT

In this paper, shape optimization is addressed through sequential quadratic programming (SQP). The recent increase in information technologies dedicated to optimal design, associated with the progress of the numerical tools, allows significant improvement in the design optimization of floating breakwaters. First of all, the physical and mechanical constraints, related to the environment of floating breakwaters, are expressed in terms of the geometrical dimensions of the latter in form of mathematical expressions. Then, the optimization procedure is developed based on SQP method and satisfactory results are obtained demonstrating the capability of this work.

Keywords: Wave modelling; floating breakwater; shape optimization; SQP; Matlab.

INTRODUCTION

Floating breakwaters present an alternative solution to conventional fixed breakwaters and can be effectively used in coastal areas with mild wave environment conditions. Poor foundation or deep-water conditions as well as environmental requirements, such as phenomena of intense shore erosion, water quality and aesthetic considerations advocate the application of such structures. They have many advantages compared to the fixed ones, e.g. absence of negative environmental impacts, flexibility of future extensions, mobility and relocation ability, lower cost and ability of a short time transportation and installation. As a result of all these positive effects, many types of floating breakwaters have been developed as described by McCartney (1985); however, the most commonly used type of floating breakwaters is the one that consists of rectangular pontoons connected to each other and moored to the sea bottom with cables or chains. Moreover, many studies have been produced on floating breakwaters (Johansson 1989; Murali and Mani, 1997; etc.), mainly concerning the wave protection improvement by different types of floating structures. Other studies have been directed towards the mooring forces and motion responses to understand the behaviour of the floating breakwaters due to sea waves (Williams and Abul-Azm,1997; Sannasiraj, 1999; and Lee 2003). Yet non of these studies have been discussing the structural design of floating breakwaters or more even optimizing its form, ignoring an essential evident, that a moored floating breakwater should be properly designed in order to ensure effective reduction of the transmitted energy and, therefore, adequate protection of the area behind it.

In fact, optimization of breakwaters has been previously discussed by Ryu and Park (2005) and focused on minimizing the cost function imposed to structural failure constraints, and also by Castillo and Mınguez (2006) for composite breakwater types and similarly concerning the minimization of initial/construction costs subjected to yearly failure rate bounds for failure modes ; where in this paper, the study is directed towards the shape optimization of floating breakwaters to reduce its weight, in accordance to the physical and mechanical constraints using the SQP. This method, one of numerous methods used in non linear programming (NLP), provides a tool to find the minimum of an objective function which depends on a set of optional free variables and is subjected to arbitrary constraints. In fact, nonlinear programming has many applications in todays engineering practice. Particularly in structural design, NLP is successfully used reducing steel weight and cost of marine structures. It is noticed that applications related to hydrodynamic aspects of offshore structures are rarely reported. This may result from several severe problems related to the hydrodynamic analysis and evaluation of offshore structures and mainly summarized in the difficulty of hydrodynamic analysis for arbitrarily shaped structures. Nevertheless, nonlinear programming is not new in designing offshore structures; for example Chou (1977) derived optimal shapes for a buoy and an ocean platform supported by four columns proposing an analytical procedure, Akagi & Ito (1984) optimized the heave motion of a hydrodynamic transparent semi submersible using a quadratic programming technique, Kagemoto (1992) optimized the arrangement of vertical floating cylinders in waves, Clauss & Birk (1996) focused on hydrodynamic shape optimization for large offshore structures (oil platforms) based on non linear programming algorithms.

The methodology followed in this paper is first identified by an analytical modelling of waves and their induced pressures and then by introducing the method of shape optimization. After this, physical and mechanical constraints concerning the floating breakwater are imposed; where the numerical analysis for mechanical constraints is based on the finite element method. Finally, a practical application with Matlab programming is developed; where it is interesting to consider the case of a breakwater appearing in ports' constructions far from the shore, at a constant depth, and at a fixed point. Then, the problems of waves propagation over a varying bathymetry and shallow water consequences are eliminated.

WAVE MODELLING

A cartesian coordinate system *Oxyz* is employed, where *Oxy* coincide with plane of the free surface at rest, *Oz* directed positive upwards, and *Ox* directed positive in the direction of propagation of the waves. The incident wave propagates in a straight line in the direction of *Ox* axe to obtain the maximum pressure applied by the waves on the breakwater (incident wave normal to the breakwater) and the movement is reduced to two dimensions (Fig 1).

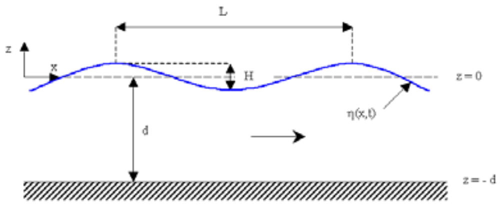

Fig.1 Wave notations

The fluid motion is defined as follows: Let t denote time, x and z the horizontal and vertical coordinates, respectively, and η the free-surface elevation above the still water level. The characteristic parameters of the wave are defined in (Fig 1). The high values of the density and sound velocity in water render the compressibility effects negligible in sea water, so it is considered incompressible. The fluid is considered also irrotational. Then, the fluid motion can be described by a velocity potential (Φ). Once the parameters characterizing the sea waves are known (Length of wave L, Period T, Height H), a model is needed to study the waves' propagations and transforms their evolution into loads on the breakwater. The well known equation, Bernoulli-Lagrange constitutes the essential equation to determine the field of wave's pressure. In general, the study of marine structures' behaviours due to waves' propagations is mostly made as part of a linear theory, where the interest in this paper is to orient the work towards the non linear approximation (Stokes 2nd order expansion). It is clear that if Φ is known throughout the fluid, the physical quantities (pressure and velocity) can be obtained from Bernoulli's equation. The boundary value problem is then defined by:

$\nabla^2\Phi = \Delta\Phi = 0$ Laplace equation in the fluid domain;

$\left(\frac{\partial\Phi}{\partial z}\right)_{z=-d} = 0$ Condition at the sea floor;

$\left(\frac{\partial\Phi}{\partial n}\right)_{x=0} = 0$ Kinematicconditionatthesolidboundary

$\left(\frac{\partial\eta}{\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x} - \frac{\partial\Phi}{\partial z}\right)_{z=\eta} = 0$ Kinematic condition at the free surface;

$\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left(\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right) + g\eta\right)_{z=\eta} = Q(t)$ Dynamic equation at the free surface.

Applying the nonlinear theory (Stockes 2nd order expansion), based on the perturbation method (Bonnefille, 1976); the expression of the pressure distribution in the case of wave-breakwater interaction, where all the waves are reflected by the breakwater (no diffraction or transmission) is given as: (Elchahal, 2006):

$$P(x,z,t) = -\rho gz + Re\left\{\frac{1}{2}\rho gH\frac{ch[k(z+d)]}{ch(kd)}\left[exp\,i(kx-\omega t) + r\,exp\,i(-kx-\omega t+\beta)\right]\right\}$$
$$+ Re\left\{\frac{3}{4}\rho gH\frac{\pi H}{L}\frac{1}{sh(2kd)}\left[\frac{ch2k(z+d)}{sh^2kd} - \frac{1}{3}\right]\left[exp\,2i(kx-\omega t) + \left(r^2+r\right)exp\,2i(-kx-\omega t+\beta)\right]\right\}$$
$$+ Re\left\{r\rho H^2\omega^2\,exp\,i(-2\omega t+\beta)\right\} - \frac{1}{4}\rho gH\frac{\pi H}{L}\frac{(r+1)}{sh(2kd)}\left[ch2k(z+d)-1\right]$$

(Where $k = 2\pi / L$ designates the wave number and ω the frequency). This repartition of the hydrodynamic pressure has a curved shape (obtained using Matlab); where its maximum is around the still water level and it decreases to zero at the top of the breakwater (with the wave height) and also decreases with water depth (Fig.2). Fixing $x = 0$ (exterior breakwater surface), and the phase angle $\beta = 0$ (vertical impermeable wall), the pressure distribution over the vertical breakwater is obtained.

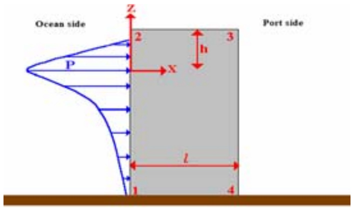

Fig. 2 Hydrodynamic pressure distribution over the breakwater

This hydrodynamic pressure is acting on the exterior surface of the breakwater due to the assumption that all the waves propagating from the ocean side are totally reflected outside the port (no transmission). It can be written as follows:

$$P = a\cosh k(z+d) + b\cosh 2k(z+d) + f \qquad (2)$$

$$a = \frac{\rho gH}{2}\frac{(r+1)}{chkd}\cos(\omega t)$$

$$b = \frac{\rho g\pi H^2}{4Lsh2kd}\left[\frac{(3r^2+3r+3)\cos(2\omega t)}{sh^2kd} - r - 1\right]$$

$$f = \frac{\rho g\pi H^2}{4Lsh2kd}\left[\left(-r^2-r-1\right)\cos(2\omega t) + r + 1\right] + \rho H^2\omega^2 r\cos(2\omega t)$$

It is reduced to an equation with hyperbolic functions of z (altitude), where the other variables independent of the altitude are collected together in the terms $a,\ b,$ and f .

SHAPE OPTIMIZATION

A moored floating breakwater should be properly designed in order to ensure: (a) effective reduction of the transmitted energy, hence adequate protection of the area behind the floating system, (b) non-failure of the floating breakwater itself and (c) non-failure of the mooring lines. The satisfaction of these 3 requirements represents the overall desired performance of the floating breakwater. The non-failure of the mooring lines has been widely studied and discussed, so the efforts in this paper are directed towards the first two issues.

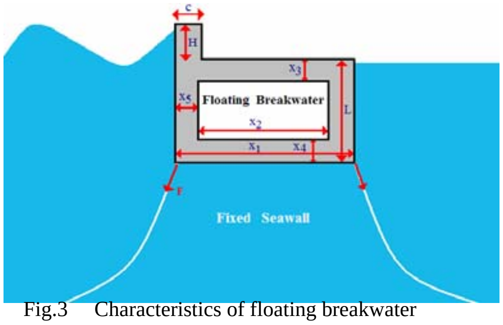

Fig.3 Characteristics of floating breakwater

The reduction of the transmitted energy is achieved by the floating breakwater itself due to a considerable depth and by the fixed seawall concept under the breakwater for the rest underwater region. Moreover, for a breakwater to float it is obviously designed with a hollow form to reduce the total weight of the structure; where such form complicates the problem and implicates more constraints to be considered during the design. Also, an additional rectangular (Fig.3) wall can be used to protect the sheltered regions from high waves; where it is sufficient to place it only from the ocean side since it has non sense to construct a rectangular breakwater with its height over the free surface level equals to a strong wave height. Then, it can be simply deduced that a floating breakwater can be assimilated to two parts: the main rectangular body possessing sufficient dimensions considering the fixed seawall concept, and a second part formed by a small rectangular wall fixed on the ocean side of the breakwater to attenuate the high waves. The dimensions of the second part are easily determined, where its height is equal to the wave height H, and its width c is taken to be 0.8 m [10].

In fact, the problem of shape optimization has been widely explored in the structural optimization area along with the rapid development of fast digital computers and numerical methods such as the finite element method. The usual shape optimization procedures start from the given initial design. The boundary of the structure is described and parameterized using a set of simple segments such as straight lines, and then the shape is varied iteratively using the information from the shape design sensitivity to achieve finally the optimal shape design. Therefore, improving the performance of floating breakwaters could open up multiple of possible uses and this because the floating breakwater, in contrary to the fixed one (the only parameter to calculate is the width being deduced from the stability condition), has many parameters characterizing its geometry and defining its shape $L, x_1, x_2, x_3, x_4, x_5$ (Fig.3). Some of these parameters are related to the same physical constraint where the rest are determined from other independent constraints, and therefore determining its geometrical dimensions cannot be performed as an ordinary calculation problem but it needs an optimisation process in order to compute these parameters taking into consideration their effects on each other. Hence, the optimisation problem is assumed to be finite dimensional constrained minimization problem, which is symbolically expressed as:

Find a design variable vector x ;

to minimize the weight function $f(x)$

subject to the *n* constraints $C_i(x) < 0$

Optimization Methodology

In constrained optimization, the general aim is to transform the problem into an easier subproblem that can then be solved and used as the basis of an iterative process. A characteristic of a large class of early methods is the translation of the constrained problem to a basic unconstrained problem by using a penalty function for constraints that are near or beyond the constraint boundary. These methods are now considered relatively inefficient and have been replaced by methods that have focused on the solution of the Kuhn-Tucker (KT) equations. The KT equations are necessary conditions for optimality for a constrained optimization problem, which can be stated as:

$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i . \nabla C_i(x^*) = 0$$

$$\lambda_i . C_i(x^*) = 0, i = 1....n$$

The solution of the KT equations forms the basis to many nonlinear programming algorithms. These algorithms attempt to compute the Lagrange multipliers directly (λ_i) . Constrained quasi-Newton methods guarantee superlinear convergence by accumulating second-order information regarding the KT equations using a quasi-Newton updating procedure. These methods are commonly referred to as Sequential Quadratic Programming (SQP) methods, since a quadratic subproblem (QP) is solved at each major iteration. A nonlinearly constrained problem can often be solved in fewer iterations than an unconstrained problem using SQP. One of the reasons for this is that, because of limits on the feasible area, the optimizer can make informed decisions regarding directions of search and step length (Arora, 2004).

The SQP implementation consists of three main stages, which are discussed briefly in the following points:

1• Updating the Hessian Matrix of the Lagrangian function:

At each major iteration a positive definite quasi-Newton approximation of the Hessian of the Lagrangian function, *H*, is calculated using the formula of BFGS method (…) where λ_i is an estimate of the Lagrange multipliers.

$$H_{k+1} = H_k + \frac{q_k q_k^T}{q_k^T s_k} - \frac{H_k^T H_k}{s_k^T H_k s_k} \quad \text{, where}$$

$$s_k = x_{k+1} - x_k$$

$$q_k = \nabla f(x_{k+1}) + \sum_{i=1}^{n} \lambda_i . \nabla C_i(x_{k+1}) - \left(\nabla f(x_k) + \sum_{i=1}^{n} \lambda_i . \nabla C_i(x_k) \right)$$

This is then used to generate a QP subproblem whose solution is used to form a search direction for a line search procedure.

2• Quadratic Programming Solution (QP)

At each major iteration of the SQP method, a QP problem of the following form is solved to obtain the search direction d :

$$\min_{d \in \Re^n} \frac{1}{2} d^T H_k d + \nabla f(x_k)^T d$$

$$\nabla C_i(x_k)^T d + C_i(x_k) = 0 \qquad i = 1, ...m_e$$

$$\nabla C_i(x_k)^T d + C_i(x_k) \le 0 \qquad i = m_e + 1, ...m$$

3• Line Search and Merit Function calculation

The solution of the QP problem is used to form a new iterate $x_{k+1} = x_k + \alpha_k d_k$, where the step length parameter α_k is determined by an appropriate line search procedure so that a sufficient decrease in a merit function is obtained.

Objective Function

The optimal solution is to design a breakwater respecting all the constraints with a minimum volume, hence the objective is to minimize the weight of the breakwater,

$$f_{ob}(x_1, x_2, x_3, x_4, x_5, F) = Lx_1 - x_2(L - x_3 - x_4) + Hc$$

Dynamic Pressure Constraint

The concept of the fixed seawall permits to determine the height of the breakwater in accordance with low hydrodynamic pressure acting on this seawall. The dynamic wave pressure is mainly concentrated near the free surface and its induced perturbation is feeble under a certain height (Fig.4); then the height of the breakwater can be limited to where the pressure is approximately unvarying corresponding to an approximate value of $P - aP\,max = 0$, where $P\max = P(z = 0)$. Finally, the height can be considered to be $L = 6m$, where this height is indeed satisfactory for a strong wave $(H = 2m)$.

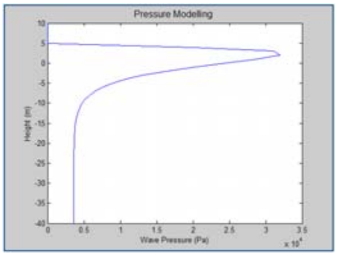

Fig.4 Wave Pressure Modelling

This constraint is independent of the other constraints, and then the height of the breakwater is determined only from it and no need to still consider the height as a variable for the rest of the optimization process.

Floating Constraint

The floating of the breakwater is a direct application of Archimedes principle where the equilibrium equation for floating can be written as: $-\rho_m(V_m + V_r)g + \rho_e V_T g = 0$, where

ρ_m and ρ_e designates the densities of the material (concrete) and the sea water respectively, V_m designates the volume of the inside material of the whole breakwater without the upper rectangular wall, V_o designates the volume of the hollow part (atmospheric pressure inside), V_r designates the volume of the upper rectangular part, where V_T designates the volume of the submerged part of the breakwater , and then $V_m + V_o = V_T$

A relation between the hollow volume and the submerged volume can be simply deduced: $V_o = \frac{\rho_m - \rho_e}{\rho_m} V_T + V_r$

The floating constraint can be expressed as follows:

$$f(x_1, x_2, x_3, x_4, x_5, F) = x_2(L - x_3 - x_4) - \frac{\rho_m - \rho_e}{\rho_m} L x_1 - V_r$$

But, really the floating constraint yields to a simple relation between the variables that can be used to reduce the number of variables in the optimization.

Stability Constraint

Stability is defined as the ability of the breakwater to right itself after being heeled over. This ability is achieved by developing moments that tend to restore the breakwater to its original condition. There are a number of calculated values that together determine the stability of a floating breakwater: 1- Initial horizontal equilibrium, 2- Heeled angle, 3- Tension in mooring lines.

First of all, this floating breakwater has a non-symmetrical shape, so initially (before any disturbance) it is necessary to maintain a horizontal equilibrium position. In this case, it can be benefited from the numerical analysis of the structure to calculate in an interesting method the new centre of gravity and then aligning it with the centre of buoyancy for the floating breakwater (Fig.6) which lies at the geometric centre of volume of the displaced water ($x_1/2$). It is based on calculating the centre of gravity of each triangle in the whole mesh triangulation process and its corresponding area instead of dividing the structure into five rectangles and writes the analytical equations of their centres (Fig.5). In fact, it is based on the basic formula of determining the centre of gravity for a structure composed from different well known determined geometrical shapes and applied for each triangle in the meshed domain.

$x_g = \frac{\sum A_i \times x_i}{\sum A_i}$ where A_i and x_i are respectively the area and the centre of gravity of the composing geometries. Then, the relevant constraint is $x_g = x_1/2$ (horizontal equilibrium condition)

$$C_2(\rho) = \frac{\sum_{i=1}^{n} A_i \times x_i}{\sum_{i=1}^{n} A_i} - \frac{x_1}{2} = 0 \qquad (5)$$

where *n* is the number of triangles in the meshed domain.

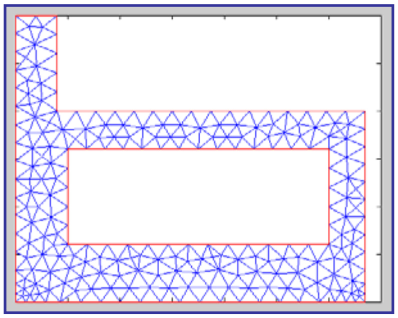

Fig. 5 Determining the centre of gravity

When the breakwater is disturbed by a wave, the centre of buoyancy moves from B to B_1 (Fig.6) because the shape of the submerged volume is changed; then the weight and the buoyancy force form a couple capable to restore the breakwater to its original position.

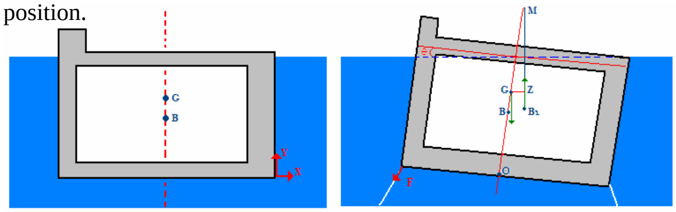

Fig.6 Stability of floating breakwater

Moreover, the distance GM known as the metacentric height illustrates the fundamental law of stability, where it must be always positive to create a restoring couple and maintain stability $G\overline{M} \geq 0$.

The equation of motion can be written as: $\sum M = I\ddot{\theta} \Rightarrow$ at equilibrium $|Mp| - M_F - M_B = 0$, where Mp is the moment of the disturbing force (wave), M_F is the moment of the tension in the mooring lines, and M_B is the moment of the buoyant fore (restoring couple). The absolute value of the disturbing moment guarantees the flexibility of the stability relation in the two senses of rotation; that is the couple produced by the weight must also be in opposite sense of the disturbing moment to be capable to right the structure to its initial position. Hence, the stability constraint can be expressed as:

$$C_3(\rho) = -W\left(\frac{x_1^2}{12L} - y_g + \frac{L}{2}\right)\sin\theta - F\cos(\alpha - \theta)x_g + F\sin(\alpha - \theta)y_g$$

$$+ \left| \int_{-L+y_g}^{0} (a\cosh k(z + d - y_g) + b\cosh 2k(z + d - y_g) + f)zdz \right.$$

$$\left. + \int_{0}^{h-yg} (a\cosh k(z + d - y_g) + b\cosh 2k(z + d + y_g) + f)zdz \right| \leq 0$$

h is the height of the breakwater portion above the still water, *α* being the angle formed by the mooring lines and the vertical (*α=20°*), and *θ* is the angle of disturbance (heeled angle); in fact it is fixed by the designer, and since the breakwater must be very rigid and stable in order to protect the ports from waves, it is taken 1.2°.(slope of 2%)

Structural constraint

This constraint constitutes a pure structural analysis of the floating breakwater, where a comprehensive numerical analysis is requested in order to determine the mechanical stresses that must be restricted to certain limits. It can be summarized by maximizing a desired property of the structure, mechanical stresses (stiffness), having a given shape. Then, the floating breakwater is modelled using the finite element method fixed on two simple supports at its bottom. It is well known that the concrete have different compression and traction limits due to its nature, and so the well known formula of Von Mises for elastic materials cannot be used. A special criterion, named the Parabolic Criteria, (Garrigues.J, 2001) mainly used for concrete is introduced in terms of the principal stresses of the breakwater and the limit stresses for the material, and is written directly in the form of optimization constraint:

$$C_4(\rho) = (\sigma_1 - \sigma_2)^2 - (\sigma_t + \sigma_c)(\sigma_1 + \sigma_2) - \sigma_t\sigma_c \leq 0 \qquad (6)$$

This constraint as the others must be computed in each iteration, which yields to solve the FEM problem in each iteration and for each new defined geometry in order to define the principal stresses. In fact, this is the heaviest constraint between the others, where in each iteration a new geometry is defined, meshed, and then passed to the pdetool to be solved.

FINITE ELEMNT METHOD

The numerical analysis of the mechanical behaviour of this floating breakwater is based on the finite element method (FEM) using the software Matlab. In fact, Matlab solve the problems of (FEM) under the partial differential equations toolbox (PDE Tool), where the mechanical problem is assimilated to an elliptic equation under the form: $-div(c \times grad(u)) + a \times u = f$ in Ω, where Ω is a bounded domain in the plane, u is the solution vector, $c,\ a\ ,f$ are complex functions defined on Ω. In structural mechanics the main problem is concentrated in solving the equilibrium equation $\overrightarrow{div\,\sigma} + \overrightarrow{f_v} = \overrightarrow{0}$ in a determined structural domain exposed to different boundary loadings (forces and displacements).

To solve this classical equilibrium equation under the elliptic family of equations, the elliptic coefficients $u,\ c,\ a\ ,f$ are defined in terms of their equivalence substitutes in a mechanical problem. The u represents the nodal displacement vector in the two directions, a equals to zero, f represents the volume forces or simply the weight $(-\rho_m \times g)$, and c stands for the matrix deduced from the stress-strain relation, assuming isotropic and isothermal conditions.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\upsilon^2} \begin{pmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & (1-\upsilon)/2 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

where σ_x and σ_y are the stresses in the x and y directions, and τ_{xy} is the *shear stress.* The material properties are expressed as a combination of E, the elastic modulus or Young's modulus, and υ Poisson's ratio.

The basic finite element procedure starts by describing the geometry of the domain Ω and the boundary conditions. The boundary conditions specify a combination between u and its normal derivative on the boundary, and are defined either under the Dirichlet form (defining displacement) or under the Neumann form (defining forces). Second, a triangular mesh is built up on the domain Ω; and finally the structure is discretized into many subregions and for each subregion the displacement field is written in terms of nodal values. The total potential energy is then minimized with respect to the nodal values to give the equilibrium relation:
$\{F\} = [k] \times \{u\}$, where $\{u\}$ is the vector of nodal displacements, $\{F\}$ is the vector of element nodal forces, and $[k]$ is the element stiffness matrix.

Once the displacement vector u is computed, it is easy to move deeper and calculate the mechanical stresses and finally the principal stresses, where these latter stresses are the one substituted in the structural constraint expression.

APPLICATION AND RESULTS

Without any further doubt, the applied method will produce a floating breakwater with a new shape providing an idea of an efficient breakwater. In this section a numerical application is developed and results are obtained based on the following numerical setup for the waves and the breakwater:

$$\text{wave}\begin{cases} L = 120m & T = 9\,sec \\ H = 2m & t = 0 \\ d = 40m & r = 0.8 \end{cases} \qquad \text{Breakwater}\begin{cases} \rho_m = 2300Kg/m^3 \\ F = 10^4\,N \\ D = 6m \end{cases}$$

The optimization problem, outlining the whole environmental conditions in floating breakwater design, is solved by the SQP method in Matlab, leading to the following results (Fig.7):

$$\begin{cases} x_1 = 6.25m, & x_2 = 5.09m, \\ x_3 = 0.3m, & x_4 = 0.66m \\ & x_5 = 0.3m, \end{cases}$$

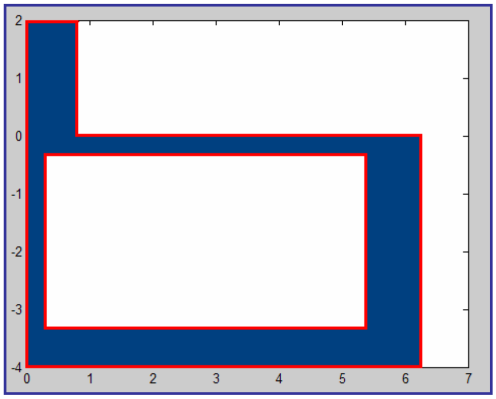

Fig.7 Floating breakwater using shape optimization

As mentioned before, each constraint (physical and mechanical) is expressed alone and in a separate program, then they are finally assembled in the optimization algorithm. These constraints are obviously written in form of mathematical equations in order to be introduced in the optimization problem. But in fact, it is not a classical optimization problem where all the constraints are only defined in terms of mathematical equations without affecting the physical significance of the latter whatever is the response in each iteration. That is, if one of the iterations reproduces a non logical solution vector, the algorithm will proceeds forward until it reach a global minimum point. But, in a problem that handles in addition to the physical constraints, a mechanical problem where the mechanical stresses has to be calculated for a new defined geometry in each iteration, any non logical iteration response will yield to arrest the optimization procedure directly without any solution. Really, in such types of problems the most important constraint is the mechanical one due to its geometrical sense, since in each iteration a new defined geometry is introduced to the problem and moreover what complicate the problem is the two geometrical shapes defined in the same domain. For example, we can simply summarize our problem as optimizing a void surface translating or moving inside another geometrical shape that also needs to be optimized. Hence, errors can occur when an iteration produces the void geometry partially outside the other geometry or intersecting with it, leading to a non meaningful geometry lacking off course the capability of meshing a non sensible geometry.

In Matlab, it is possible to overcome such difficulties by introducing a conditional algorithm in the Matlab programs to sustain the execution of the rest of the optimization iterations although one of them or more falls in such error. This can be done by ignoring any non sensible defined geometry, based on the failure message deduced from the meshing commands, by providing a large stress tensor value $(\sigma = 9MPa)$; forcing the optimization procedure to skip directly to another iteration without executing the rest of the finite element procedure in the previous one; which forces the optimization algorithm to continue since the mechanical constraint is not satisfied at all. Also, this problem can not be treated by adjusting appropriate upper and lower bounds of the inside rectangle with respect to the larger one, since all these values are varying and we cannot introduce variables inside the bounds vectors.

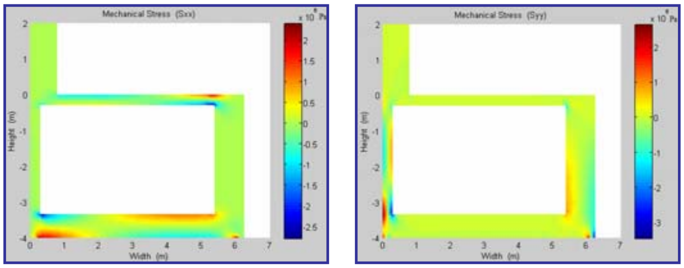

Figure 8 Mechanical stresses σ_x (left) and σ_y (right)

In consequence to our applied method, it is apparent that we ended up with a very logical and accepted solution to our problem considering an overall breakwater problem. In fact, it is not only a problem of volume consuming, but also a structural advantage where the floating breakwater is working approximately in the same stress domain (Fig.8); while in the case of a fixed bottom breakwater (filled material breakwater) the stress domain is largely varying between the points inside the breakwater. This is an additional advantage for the floating breakwater, since the more the inside points are working on closer stresses values the more the extended life of the structure is expected and vice versa. Moreover, we can notice (Fig. 8) the respected limits of the mechanical stresses due to the imposed structural constraints, where the concrete has its traction and compression limits as follows: $\sigma_t = 4MPa$, $\sigma_c = -40MPa$.

CONCLUSION

Further to the previous discussions, SQP has been utilized for optimizing floating breakwaters, where its initial shape has been expressed as a function of the design variables, or simply the geometrical dimensions, in order to implement the optimization problem. This work constitutes a comprehensive study on floating breakwaters, since it commenced with modelling sea waves and determining its induced pressures on structures. Second, it considered the physical constraints such as floating and stability, where this latter constraint is mainly derived from the sea wave's pressure; also the essential constraint in structures design represented by limitations of the mechanical stresses in not omitted. Finally, all the preceding constraints are accumulated in a shape optimization problem resulting in a resistant floating breakwater.

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