Numerical Analysis of Thermo-Elastic Damage of a Cylindrical Body: Application to Engine System

Presented by : **Bilal Taher**

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Authors:

Bilal Taher SET, UTBM, Belfort, France

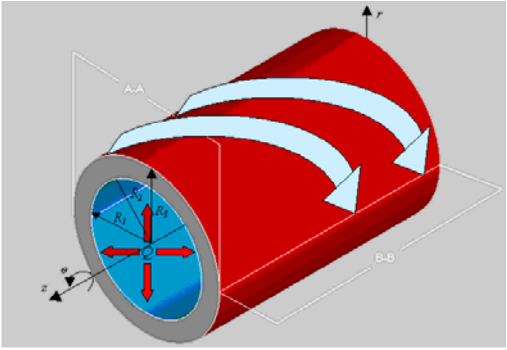
Said Abboudi SET, UTBM, Belfort, France

Rafic Younes 3M, ULFG, Beirut, Lebanon

Introduction

- **Most systems or mechanical components are subject to a thermo-mechanical coupling (for example the cylinder of engine).**
- **Stress < elastic limit stress → Thermo-elastic behavior must be applied.**
- **Variation and periodicity of the heat flux → Oligocyclic fatigue of material.**
- **Main existing laws of damage [Lemaitre & Chaboche] are applied only to mechanical stress**
- **we are interested to calculate the thermo mechanical damage in each point using numerical method**

General description

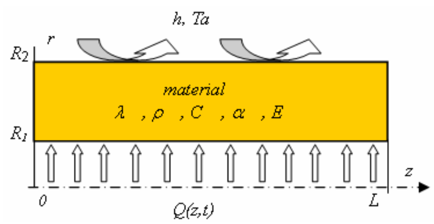


- The model is a hollow cylinder of Steel.

- The study is done in 2D at the B-B section, because in the section A-A, we have a symmetric charge.

General description

One section of this cylinder:



Hypotheses:

- Isolation and fixing at z = 0, z = L

$U = 0, W = 0$

- Variable heat flux Q(z,t) at r = R1

- Convection at r = R2

- This cylinder is constituted of steel it has these physical properties

	Steel
ρ: density	7850 kg/m3
c: heat capacity	435 J/Kg.°K
λ: thermal conductivity	60.5 W/m°K
E: young modulus	210 GPa
ν: poison coefficient	0.29
L: length	0.1m
e: thickness	e_2 = 0.012 m

Thermal Formulation

- First thermodynamic principal

$$\rho C\dot{T} = div(\lambda \,\overrightarrow{grad}\, T) + \sigma : \dot{\varepsilon}^p - A_k\dot{V}_k + \dot{r} + T\left[\frac{\partial \sigma}{\partial T} : \dot{\varepsilon}^e + \frac{\partial A_k}{\partial T}\dot{V}_k\right]$$

- The stress in the body is < the elastic stress limit $\sigma < \sigma_y$

- In this study: the non recoverable energy, the plastic deformation, the variation of stress and the internal sources are neglected

$$A_k\dot{V}_k = 0 \qquad T\frac{\partial A_k}{\partial T}\dot{V}_k = 0 \qquad \dot{r} = 0 \qquad \sigma : \dot{\varepsilon}^p = 0 \qquad T\frac{\partial \sigma}{\partial T} : \frac{\partial \varepsilon^e}{\partial t} = 0$$

- Under these hypotheses, the thermal model of the studied system is

$$(\rho C)\cdot\frac{\partial T}{\partial t} = \lambda\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial z^2}\right)$$

- This temperature is calculated numerically by the discretization of this equation using the finite difference method

Thermo-mechanical Formulation

- Equilibrium of force

$$\begin{cases} \dfrac{\partial \sigma_{rr}}{\partial r} + \dfrac{\partial \sigma_{zr}}{\partial z} + \dfrac{\sigma_{rr} - \sigma_{\theta\theta}}{r} = 0 \\ \dfrac{\partial \sigma_{rz}}{\partial r} + \dfrac{\partial \sigma_{zz}}{\partial z} + \dfrac{\sigma_{rz}}{r} = 0 \end{cases}$$

- The general Hooke's law is $\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda e - \beta\tau)\delta_{ij}$

- The replacement of Hooke's law in the equilibrium of force, gives the following equation

$$\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2} + \frac{1}{1-2\nu}\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} - \frac{u_r}{r^2} + \frac{\partial^2 u_z}{\partial r \partial z}\right) - 2\frac{1+\nu}{1-2\nu}\alpha\frac{\partial \tau}{\partial r} = 0$$

$$\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{1-2\nu}\left(\frac{\partial^2 u_r}{\partial z \partial r} + \frac{1}{r}\frac{\partial u_r}{\partial z} + \frac{\partial^2 u_z}{\partial z^2}\right) - 2\frac{1+\nu}{1-2\nu}\alpha\frac{\partial \tau}{\partial z} = 0$$

- The thermo-mechanical displacements are calculated numerically by the discretization of this equation with the finite difference method

Damage Model

- Ncd is the number of cycles to damage, it is calculated when D = 1, using wohler Miner formulation

$$\frac{\delta D}{\delta N} = \frac{\sigma_M - \sigma_l(\bar{\sigma})}{\sigma_u - \sigma_M}\left(\frac{\sigma_M - \bar{\sigma}}{B(\bar{\sigma})}\right)^{\beta}$$

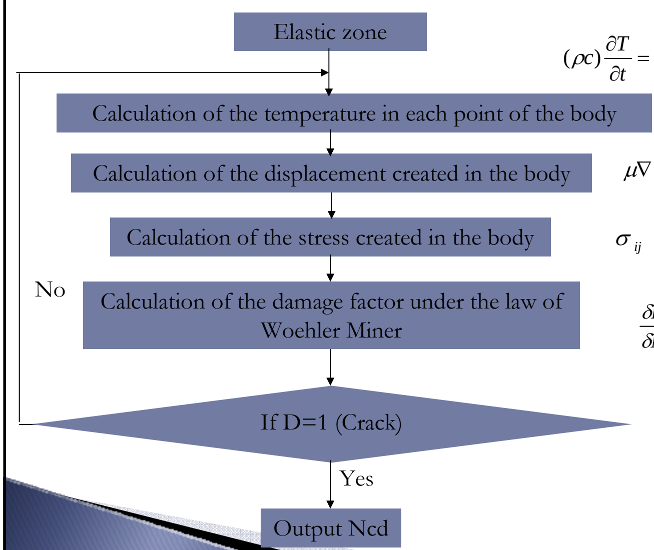
Where: $\sigma_l(\bar{\sigma}) = \bar{\sigma} + \sigma_{l_0}\left(1 - \frac{\bar{\sigma}}{\sigma_u}\right)$ $\quad B(\bar{\sigma}) = B_0\left(1 - \frac{\bar{\sigma}}{\sigma_u}\right)$

$\sigma_M , \bar{\sigma}$ are the maximum and average constraints applied

- The parameters of this law are shown in this table

Material	σ_u (MPa)	σ_l (MPa)	B_0 (MPa)	β
Steel	360	2005	6320	3.3

Flow-chart of numerical calculation



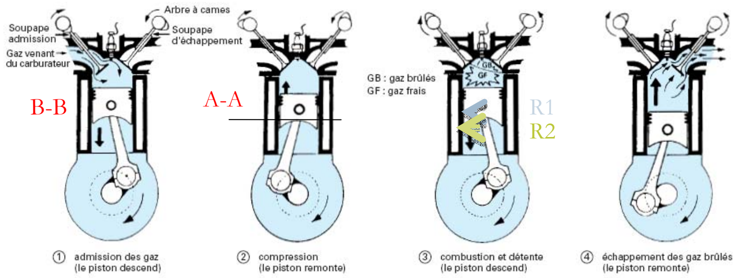
$$(\rho c)\frac{\partial T}{\partial t} = \frac{1}{r}\frac{\partial}{\partial r}(\lambda\frac{\partial T}{\partial r}) + \frac{\partial T}{\partial z}(\lambda\frac{\partial T}{\partial z})$$

$$\mu\nabla^2 u_i + (\lambda + \mu)u_{k,ki} - \beta\tau_{,i} = 0$$

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + (\lambda e - \beta\tau)\delta_{ij}$$

$$\frac{\delta D}{\delta N} = \frac{\sigma_M - \sigma_l(\bar{\sigma})}{\sigma_u - \sigma_M}\left(\frac{\sigma_M - \bar{\sigma}}{B(\bar{\sigma})}\right)^{\beta}$$

Description of the application

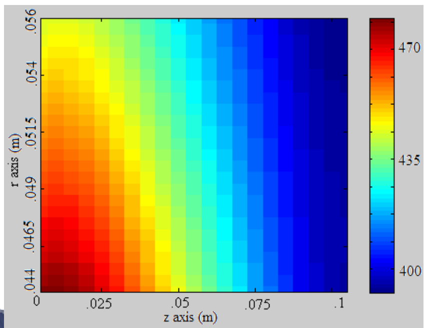


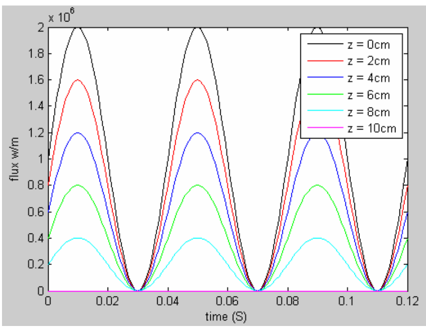
- **Simulation of heat flux is very complex, we can approximate this heat flux by a function similar to that of a 4 times engine.**
- **Variable heat flux at r = R1:** $Q(z,t) = Q_0(1 - z/L)(1 + \sin(\omega t))$

  Where: $Q_0 = 1 \rightarrow 2\, MW/m^2$ $N = 250 \rightarrow 3000\ cycle/\min$
- **The cooling convection at r = R2:** $h = 2000 \rightarrow 4000\ W/(m^2.K)$

Thermal analysis

- **Temperature is distributed in a hyperbolic isothermal field having as a center the PMH of the cylinder**
- **Temperature decrease in the r and z direction on the convection face and on the (PMB)**
- **The applied heat flux is shown in this figure**

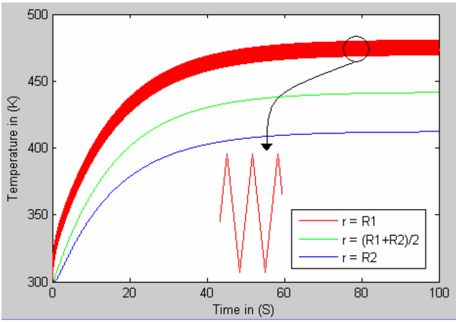


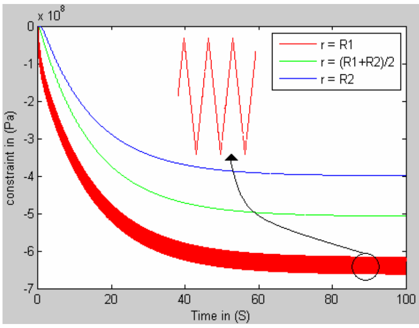


At time t = 80

Thermomechanical analysis

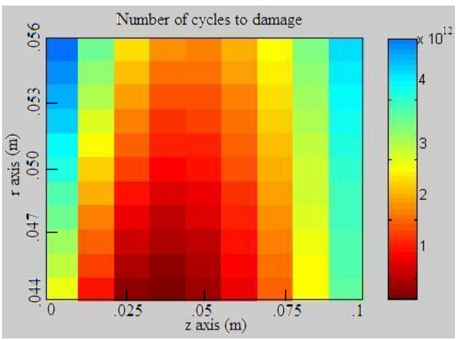
- The maximum temperature and stress are located at $r = R_1$ (high temperature)
- The variation of the stress disappears when the oscillation of the temperature disappears (r = (R1+R2)/2 & r = R2)





Dammage analysis

- Ncd is calculated numerically
- Each finite element represents the number of cycles to damage and the location of this damage
- The higher Ncd appears firstly at $r = R_1$, $z = L/3$, with the higth temperature and stress applied in the internal site.



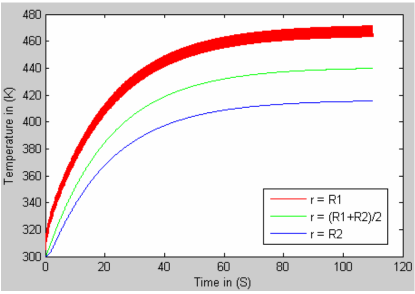
utbm

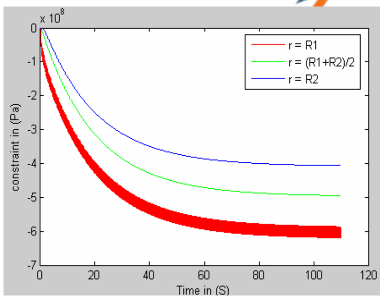
Results

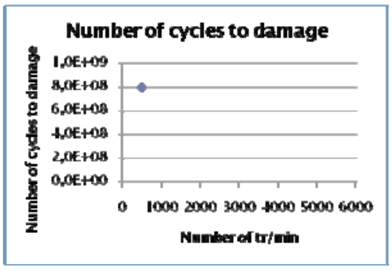
SET

For N = 480 rd/min, Q_0= $5x10^5$

h = 2000 w/m^2K







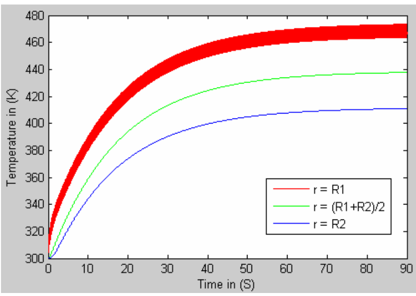
utbm

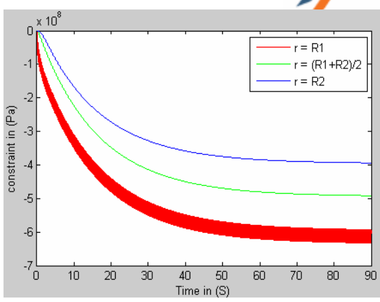
Results

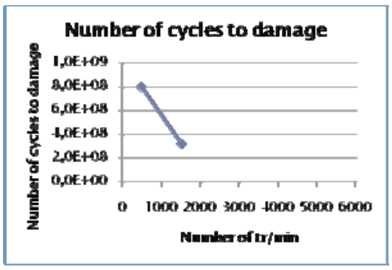
SET

For N = 1500 rd/min, Q_0= $5.5x10^5$

h = 2300 w/m^2K



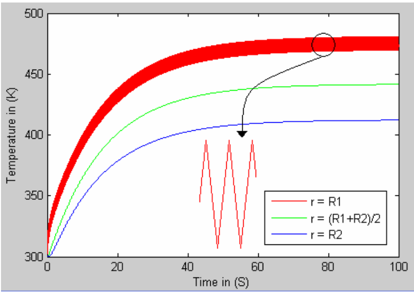


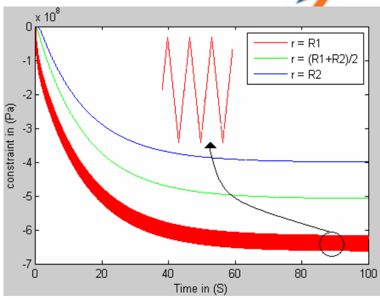


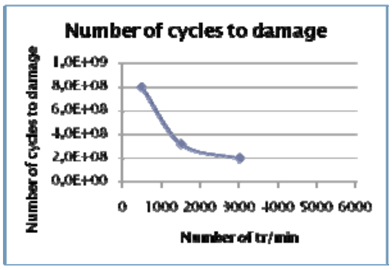
Results

For N = 3000 rd/min, Q_0= 6x10^5

h = 2500 w/m^{2}K



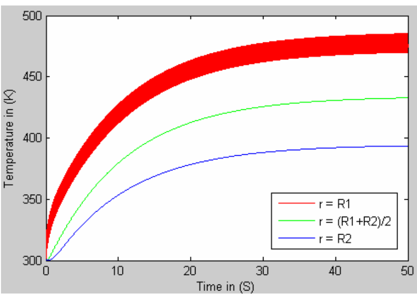


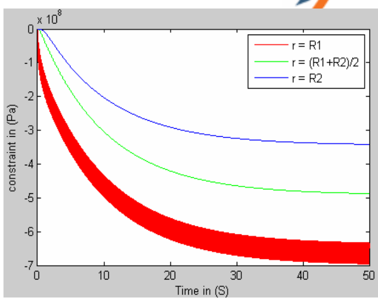


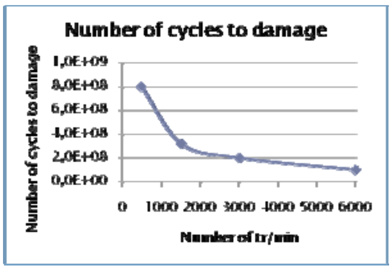
Results

For N = 6000 rd/min, Q_0= 8x10^5

h = 4000 w/m^{2}K

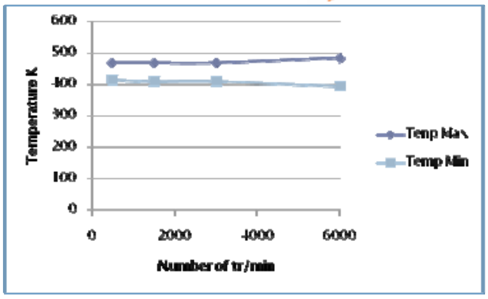




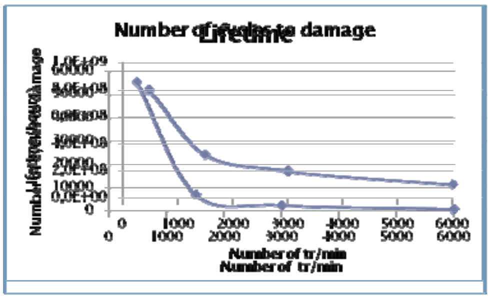


Discussion of results

- This Figure shows the values of the inner (*r=R1*) and outer (*r=R2*) temperatures in the body z = L/3 (approximately constant).



- This figure shows the value of the Ncd as a function of frequency N (tr/min).
- Although the temperature is constant Ncd changes with the frequency N



- High frequency N → High fatigue so small Ncd.
- Low Ncd → Small lifetime.
- To maximize the lifetime of the cylinder, we have to use low frequencies N as possible .

Conclusions and perspectives

Conclusions

- We presented the results of our study of the damage in each finite element of the body in thermo-elastic regime.
- This distribution of the damage permits to reinforce the cylinder with different methods in order to obtain the maximum lifetime.
- The frequency N of the heat flux has an important impact despite the use of the same temperature and stress.
- Also, it is perfect to rum the engine with the lower value of frequency possible

Perspectives

- Study the damage in each finite element of the cylinder in the thermo-elasto-plastic regime.

Thank you for your attention