

## **Modelling and Optimizing Floating Breakwaters Using Density Distribution**

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### **ABSTRACT**

In this paper, structural optimization is addressed through a density distribution process. The recent increase in information technologies dedicated to optimal design, associated with the progress of the numerical tools, allows significant improvement in the design optimization of mechanical structures. First of all, the geometrical domain of a floating breakwater is discretized into small triangles and each one is represented by the corresponding element in the density vector. Second, the constraints, related to the environmental field of floating breakwaters, have been expressed in terms of this density vector to be expressed later in an optimization problem. Finally, the optimization procedure is developed under the genetic algorithms and satisfactory results are obtained demonstrating the capability of our method.

**KEYWORDS:** Wave modelling; floating breakwater; optimization; Matlab.

### **INTRODUCTION**

Floating breakwaters present an alternative solution to conventional fixed breakwaters and can be effectively used in coastal areas with mild wave environment conditions. Poor foundation or deep-water conditions as well as environmental requirements, such as phenomena of intense shore erosion, water quality and aesthetic considerations advocate the application of such structures. They have many advantages compared to the fixed ones, e.g. absence of negative environmental impacts, flexibility of future extensions, mobility and relocation ability, lower cost and ability of a short time transportation and installation. As a result of all these positive effects, many types of floating breakwaters have been developed as described by McCartney (1985); however, the most commonly used type of floating breakwaters is the one that consists of rectangular pontoons connected to each other and moored to the sea bottom with cables or chains (Loukogeorgaki and Angelides 2005). Moreover, many studies have been produced on floating breakwaters (Johansson 1989; Murali and Mani, 1997; etc.), mainly concerning the wave protection improvement by different types of floating structures. Other studies have been directed towards the mooring forces and motion responses to understand the behaviour of the floating breakwaters due to sea waves (Williams and Abul-Azm, 1997; Sannasiraj, 1999; and Lee 2003). Yet non of these studies have been discussing the structural design of floating breakwaters or more even optimizing its form, ignoring an essential evident, that a moored floating breakwater should be properly designed in order to ensure effective reduction of the transmitted energy and, therefore, adequate protection of the area behind it.

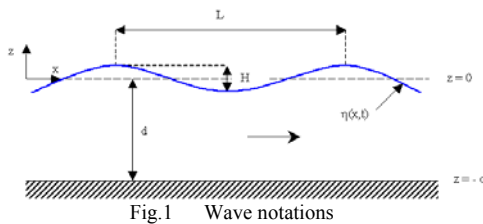
In fact, optimization of breakwaters has been previously discussed by Ryu and Park (2005) and focused on minimizing the cost function imposed to structural failure constraints, and also by Castillo and Minguez (2006) for composite breakwater types and similarly concerning the minimization of initial/construction costs subjected to yearly failure rate bounds for failure modes; where in this paper, the study is directed towards optimization of floating breakwaters to reduce its weight, or to represent a new form, in accordance to the physical and mechanical constraints using a density distribution vector. But it is good to declare that there is some work spent on shape optimization in ocean field but for offshore structures only; for example Chou (1977) derived optimal shapes for a buoy and an ocean platform supported by four columns proposing an analytical procedure, Akagi & Ito (1984) optimized the heave motion of a hydrodynamic transparent semi submersible using a quadratic programming technique, Kagemoto (1992) optimized the arrangement of vertical floating cylinders in waves, Claus & Birk (1996) focused on hydrodynamic shape optimization for large offshore structures (oil platforms) based on non linear programming algorithms.

The optimization and mainly the topology optimization generates the optimal shape of a mechanical structure by representing a new mass distribution; it is interesting to briefly review some related works in this field to evaluate our proposed method among the others applied in this domain. It is mainly based on two approaches: An approach introduced by Bendsoe and Kikushi (1988) is that of homogenization, it consists in dealing with a continuous density of material. In the end of this deterministic optimization, the current density is forced toward value 1 or 0, that respectively stands for material presence or absence and it is limited only to the linear elasticity case. Moreover, it cannot address loading that apply on the actual boundary of the shape to be determined and hardly handles optimization for multiple loadings. Another approach to topology design is that of stochastic optimization, such as involved in simulated annealing and genetic algorithms. The GA methods have been applied to topological optimization by Jensen (1992), Chapman (1994) and Kane (1995); but what attracted us is optimization based on GA due to its powerful strength in dealing with problems with large number of variables as the case of topology optimization. They have gradually been recognized as powerful stochastic optimization algorithms far away since the seminal work of Holland (1975) and the comprehensive study of Goldberg. Their strength proceeds from their wide range of applications: GA can handle non derivable, non continuous and even non analytically defined functions. For these reasons they can be used in problems for which there are no data on the possible solution; hence they are useful in some irregular or special kinds of problems including ours.

The methodology followed in this paper is first identified by an analytical modelling of waves and their induced pressures and then by introducing the method of topology optimization. After this, the physical and mechanical constraints concerning floating breakwaters are imposed; where the numerical analysis for mechanical constraints is based on the finite element method. Finally, a practical application with Matlab programming is developed; where it is interesting to consider the case of a breakwater appearing in ports' constructions far from the shore, at a constant depth, and at a fixed point.

## WAVE MODELLING

A cartesian coordinate system  $Oxyz$  is employed, where  $Oxy$  coincide with plane of the free surface at rest,  $Oz$  directed positive upwards, and  $Ox$  directed positive in the direction of propagation of the waves. The incident wave propagates at a straight line in the direction of  $Ox$  axe to obtain the maximum pressure applied by the waves on the breakwater (incident wave normal to the breakwater) and the movement is reduced to two dimensions (Fig 1).



The fluid motion is defined as follows: Let  $t$  denote time,  $x$  and  $z$  the horizontal and vertical coordinates, respectively, and  $\eta$  the free-surface elevation above the still water level. The characteristic parameters of the wave are defined in (Fig 1). The high values of the density and sound velocity in water render the compressibility effects negligible in sea water, so it is considered incompressible. The fluid is considered also irrotational. Then, the fluid motion can be described by a velocity potential ( $\Phi$ ). Once the parameters characterizing the sea waves are known (Length of wave  $L$ , Period  $T$ , Height  $H$ ), a model is needed to study the waves' propagations and transforms their evolution into loads on the breakwater. The well known equation, Bernoulli-Lagrange constitutes the essential equation to determine the field of wave's pressure. In general, the study of marine structures' behaviours due to waves' propagations is mostly made as part of a linear theory, where the interest in this paper is to orient the work towards the non linear approximation (Stokes 2<sup>nd</sup> order expansion). It is clear that if  $\Phi$  is known throughout the fluid, the physical quantities (pressure and velocity) can be obtained from Bernoulli's equation. The boundary value problem is then defined by:

$$\begin{aligned} \nabla^2 \Phi &= \Delta \Phi = 0 && \text{Laplace equation in the fluid domain;} \\ \left( \frac{\partial \Phi}{\partial z} \right)_{z=-d} &= 0 && \text{Condition at the sea floor;} \\ \left( \frac{\partial \Phi}{\partial n} \right)_{x=0} &= 0 && \text{Kinematic condition at the solid boundary;} \\ \left( \frac{\partial \eta}{\partial t} + \frac{\partial \Phi}{\partial x} \frac{\partial \eta}{\partial x} - \frac{\partial \Phi}{\partial z} \right)_{z=\eta} &= 0 && \text{Kinematic condition at the free surface;} \\ \left( \frac{\partial \Phi}{\partial t} + \frac{1}{2} \left( \left( \frac{\partial \Phi}{\partial x} \right)^2 + \left( \frac{\partial \Phi}{\partial z} \right)^2 \right) + g\eta \right)_{z=\eta} &= Q(t) && \text{Dynamic equation at} \end{aligned}$$

the free surface.

Applying the nonlinear theory (Stokes 2<sup>nd</sup> order expansion), based on the perturbation method (Bonnefille, 1976); the expression of the

pressure distribution in the case of wave-breakwater interaction, where all the waves are reflected by the breakwater (no transmission of waves is considered) is given as: (Elchahal, 2006):

$$\begin{aligned} P(x, z, t) &= -\rho g z + \text{Re} \left\{ \frac{1}{2} \rho g H \frac{\text{ch}[k(z+d)]}{\text{ch}(kd)} \right. \\ &\quad \left. \left[ \exp i(kx - \omega t) + r \exp i(-kx - \omega t + \beta) \right] \right\} \\ &+ \text{Re} \left\{ \rho H^2 \omega^2 \exp i(-2\omega t + \beta) \right\} - \frac{1}{4} \rho g H \frac{\pi H}{L} \frac{(r+1)}{\text{sh}(2kd)} [ch 2k(z+d) - 1] \\ &+ \text{Re} \left\{ \frac{3}{4} \rho g H \frac{\pi H}{L} \frac{1}{\text{sh}(2kd)} \left[ \frac{ch 2k(z+d)}{\text{sh}^2 kd} - \frac{1}{3} \right] \right. \\ &\quad \left. \left[ \exp 2i(kx - \omega t) + (r^2 + r) \exp 2i(-kx - \omega t + \beta) \right] \right\} \end{aligned} \quad (1)$$

(Where  $k = 2\pi/L$  designates the wave number,  $\omega$  the frequency, and  $r$  the reflection coefficient). This repartition of the hydrodynamic pressure has a curved shape (obtained using Matlab); where its maximum is around the still water level and it decreases to zero at the top of the breakwater (with the wave height) and also decreases with water depth (Fig.2). Fixing  $x = 0$  (exterior breakwater surface), and the phase angle  $\beta = 0$  (vertical impermeable wall), the pressure distribution over the vertical breakwater is obtained.

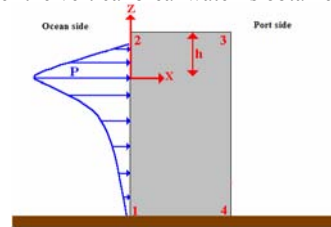


Fig.2 Hydrodynamic pressure distribution for a bottom fixed breakwater

This hydrodynamic pressure is acting on the exterior surface of the breakwater due to the assumption that all the waves propagating from the ocean side are totally reflected outside the port (no transmission). It can be written as follows:

$$\begin{aligned} P &= a \cosh k(z+d) + b \cosh 2k(z+d) + f \quad (2) \\ a &= \frac{\rho g H}{2} \frac{(r+1)}{\text{ch} kd} \cos(\omega t) \quad b = \frac{\rho g \pi H^2}{4L \text{sh} 2kd} \left[ \frac{(3r^2 + 3r + 3) \cos(2\omega t)}{\text{sh}^2 kd} - r - 1 \right] \\ f &= \frac{\rho g \pi H^2}{4L \text{sh} 2kd} \left[ (-r^2 - r - 1) \cos(2\omega t) + r + 1 \right] + \rho H^2 \omega^2 r \cos(2\omega t) \end{aligned}$$

It is reduced to an equation with hyperbolic functions of  $z$  (altitude), where the other variables independent of the altitude are collected together in the terms  $a$ ,  $b$ , and  $f$ .

## OPTIMIZATION

A moored floating breakwater should be properly designed in order to ensure effective reduction of the transmitted energy and, therefore, adequate protection of the area behind the floating system. This design objective is subjected to the following constraints: (a) effective reduction of the transmitted energy, hence adequate protection of the area behind the floating system, (b) non-failure of the floating breakwater itself and (c) non-failure of the mooring lines. The satisfaction of these 3 requirements represents the overall desired performance of the floating breakwater (Loukogeorgaki and Angelides 2005). The non-failure of the mooring lines has been widely studied and discussed, so the efforts in this paper are directed towards the first two issues. Moreover, for a breakwater to float, it is obviously designed with a hollow form to reduce the total weight of the structure; where such form complicates the problem and implicates more constraints to

be considered during the design. The reduction of the wave energy is achieved by the floating breakwater itself due to a considerable depth (depth of the floating breakwater) and by the fixed seawall concept under the breakwater for the rest underwater region. This seawall concept is derived from fluid mechanics theories; it is evident that the wave is very weak near the seabed, hence the velocity of its particles is relatively small and it increases in magnitude till it reaches the free surface. Then, it can be considered that up to a certain depth the wave effects are weak and can be omitted. In fact, it is just an approach to simplify the problem of wave structure interaction and to go forward in studying the structural requirements of such floating breakwaters. Practically, it belongs to principality of Monaco harbour, where this seawall works by stimulating the inertia of the mass of water between the lower side of the caisson and the sea bed, which then reacts like a real wall against which the surge is reflected.

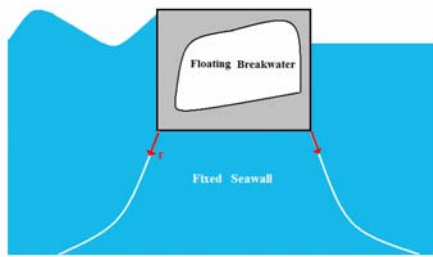


Fig.3 Characteristics of floating breakwater

Topology optimization is a much more flexible design tool than classical structural shape optimization, where in the latter only a selected part of the boundary is varied without any chance to generate a lightness hole, for example. Also, in topology all the domain is under optimization, and hence a wide range of solutions can be expected. Thus, improving the performance of floating breakwaters could open up multiple of possible cases and this because the floating breakwater, in contrary to the fixed one (the only parameter to calculate is the width being deduced from the stability condition), has many parameters characterizing its geometry (Fig.3). Some of these parameters are determined from a physical constraint where the rest are determined from other independent constraints, and therefore determining its topology cannot be performed as an ordinary calculation problem but it needs an optimisation process in order to compute these parameters taking into consideration their effects upon each other. Moreover, the optimization efforts are directed towards the mass distribution inside the rectangular breakwater, since the shape optimization is not of great importance in this problem where the external rectangular form is very effective for the case of floating breakwaters. Therefore, the optimisation problem is assumed to be finite dimensional constrained minimization problem, which is symbolically expressed as:

Find a design variable vector (binary density vector)  $\rho$  ;  
to minimize the weight function  $f_{ob}(\rho)$   
subjected to the  $n+m$  constraints  $C_i(\rho) < 0, \quad i = 1, \dots, n$   
 $G_j(\rho) = 0, \quad j = 1, \dots, m$

### Optimization Procedure

The work introduced in this paper, comprising topology optimization design with density distribution, uses genetic algorithms and is based on binary representation. It relies on dividing the design domain into a finite number of unequal random triangles. This discretization operation is executed by an arbitrary triangular mesh generation for the design domain based on the Delaunay Triangulation method in the Matlab. The number of triangles indicates the total number of variables for the optimization problem; where the latter is initially

indicated and controlled by us upon choosing the appropriate triangular mesh form (Fig. 4). It is important to note that this triangular mesh is not the same one used for computing the mechanical stresses, but it is just a technical operation for dividing the geometrical shape into small finite shapes to be easily defined in the optimization process. This importance appears in differentiating between the size of the individuals (the density vector used to encode a structure) and the size of the mesh for the mechanical computation, and by this way we can use very fine meshes without affecting the scale of the general problem. After this, a density vector is created having the same length as the number of meshing triangles in the design domain and holding only the values 0 or 1 corresponding to filled or void triangles; where this latter describes the density distribution inside the studied domain.

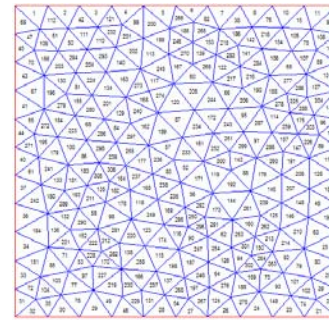


Figure 4 Domain Discretization

The index associated to each element in the density vector,  $\rho$ , represents the density of the triangle having the same index, where it creates the relation between the geometrical identification of this triangle, its location inside this domain, and its value in the density vector. This labelling or numbering is an arbitrary process where adjacent bits in the bit string representation do not necessarily correspond to neighbour elements of the domain.

$$\rho = (\rho_1, \rho_2, \dots, \rho_n)$$

The interest in this problem lies in the geometry description where it can be written or expressed in terms of triangles, which in their term are expressed in terms of their corresponding densities giving them the ability of presence or absence. In fact, this significance is not only limited to the expression of complicated and arbitrary geometries in mathematical formulas, but also in the control of keeping or removing boundary segments in the problem.

After defining the optimization procedure, a Matlab program is developed to define in each iteration a new density vector defining a new corresponding geometrical structure. This new structure is the one passed for the mechanical behaviour study, based on the finite element method, and the rest of the optimization constraints. This program is developed in conjunction of the GA toolbox in Matlab.

### Objective Function

The optimal solution is to design a breakwater respecting all the constraints with a minimum volume, hence the objective is to minimize the weight of the breakwater. Since the geometry of the structure is expressed in terms of the density distribution or mesh triangulation, the weight will be expressed in terms of the latter.

$$f_{ob}(\rho) = \rho_m \sum_{i=1}^n \rho_i \times A_i \quad (3)$$

where  $n$  is the number of triangles,  $\rho_m$  is the density of the material,  $\rho_i$  and  $A_i$  are the densities and areas of the corresponding triangles. In this way the complicated geometrical form or its arbitrary distribution is simply expressed by this simple formula, since the

presence or absence of each triangle in the weight calculation is guaranteed by its corresponding density value in the density vector.

### Dynamic Pressure Constraint

The concept of the fixed seawall permits to determine the height of the breakwater in accordance with low hydrodynamic pressure acting on this seawall. The dynamic wave pressure is mainly concentrated near the free surface and its induced perturbation is low under a certain height (Fig.5); then the height of the breakwater can be limited to where the pressure is approximately unvarying corresponding to an approximate value of  $P - aP_{max} = 0$ , where  $P_{max} = P(z = 0)$ . Finally, the height can be considered to be  $D = 6m$ , where this height is indeed satisfactory for a strong wave ( $H = 2m$ ).

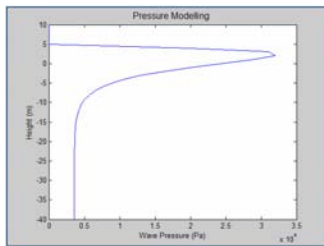


Fig.5 Wave Pressure Modelling

This constraint is independent of the other constraints, and then the height of the breakwater is determined only from it and no need to still consider the height as a variable for the rest of the optimization process.

### Floating Constraint

The floating of the breakwater is a direct application of Archimedes principle where the equilibrium equation for floating can be written as:  $-\rho_m V_m g + \rho_e V_T g = 0$ , where  $\rho_m$  and  $\rho_e$  designates the densities of the material (concrete) and the sea water respectively,  $V_m$  designates the volume of the inside material of the whole breakwater, where  $V_T$  designates the volume of the submerged part of the breakwater. The occupied volume of the breakwater can be simply expressed in terms of the areas and densities of the meshing triangles (transform the volume notations into surface notations,  $S_T$ , per 1m length).

$$\sum_{i=1}^n \rho_i \times A_i \leq S_T \frac{\rho_e}{\rho_m}$$

A relation between the hollow volume and the submerged volume can be directly expressed, in terms of the densities of the meshing triangles, as the floating constraint.

$$C_1(\rho) = \sum_{i=1}^n \rho_i \times A_i - S_T \frac{\rho_e}{\rho_m} \leq 0 \quad (4)$$

### Stability Constraint

Stability is defined as the ability of the breakwater to right itself after being heeled over. This ability is achieved by developing moments that tend to restore the breakwater to its original condition. In this case, the notation of stability is restricted to static stability only, where it is important to determine whether the breakwater will right itself due to a disturbing wave under a defined tension in the mooring lines and under a maximum limiting angle of roll; hence no dynamic equations will be included in this study. There are a number of calculated values that together determine the stability of a floating breakwater: 1- Initial horizontal equilibrium, 2- Heeled angle, 3- Tension in mooring lines. First of all, this floating breakwater has a rectangular shape with an arbitrary core, so initially (before any disturbance) it is necessary to

maintain a horizontal equilibrium position. In this case, it can be benefited from the numerical analysis of the structure to calculate in an interesting method the new centre of gravity variables and then aligning it with the centre of buoyancy of the floating breakwater (Fig.6) which lies at the geometric centre of volume of the displaced water ( $x_1/2$ ). In fact, the calculation is based on the basic formula of determining the centre of gravity for a structure composed from different well known determined geometrical shapes.

$$x_g = \frac{\sum A_i \times x_i}{\sum A_i} \text{ where } A_i \text{ and } x_i \text{ are respectively the area and the}$$

centre of gravity of the composing geometries.

But, in such problems where the geometry is taking different shapes and varying its topology in each iteration, it will be impossible to calculate the centre of gravity in the traditional or analytical methods. Benefiting from various tools and functions in Matlab, the centre of gravity and area of each triangle are calculated in the whole mesh triangulation domain including both filled and void triangles. Then, we multiply their product by the density vector excluding in this manner all the void triangles from the real calculation of the centre of gravity. It is important to note that the areas in the denominator are also multiplied by the density vector to include only the filled material triangles in calculation.

$$x_g = \frac{\sum_{i=1}^n \rho_i \times A_i \times x_i}{\sum_{i=1}^n \rho_i \times A_i} \quad \text{and} \quad y_g = \frac{\sum_{i=1}^n \rho_i \times A_i \times y_i}{\sum_{i=1}^n \rho_i \times A_i}$$

where  $x_i$  and  $y_i$  are the coordinates of the centre of gravity of each triangle. Then, the relevant horizontal stability constraint ( $x_g = x_1/2$ ) is written as follows:

$$G_1(\rho) = \frac{\sum_{i=1}^n \rho_i \times A_i \times x_i}{\sum_{i=1}^n \rho_i \times A_i} - \frac{x_1}{2} = 0 \quad (5)$$

Second, when the breakwater is disturbed by a wave, the centre of buoyancy moves from B to B<sub>1</sub> (Fig.6) because the shape of the submerged volume is changed; then the weight and the buoyancy force form a couple capable to restore the breakwater to its original position. The weight of the breakwater can be expressed in terms of the density

vector as the objective function:  $W = \rho_m \times \sum_{i=1}^n \rho_i \times A_i$

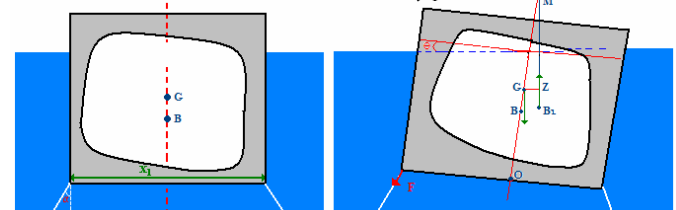


Fig.6 Stability of floating breakwater

Moreover, the distance GM known as the metacentric height illustrates the fundamental law of stability, where it must be always positive to create a restoring couple and maintain stability  $\overline{GM} \geq 0$ .

Finally, stability is achieved by the restoring couple (weight-buoyancy) and by the tension in the mooring lines. This stability is determined around the centre of gravity, hence the moments developed by the restoring couple and the tension in cables must equilibrate the moment derived from the incoming waves.

$|Mp| - M_F - M_B = 0$ , where  $Mp$  is the maximum value of the moment of the disturbing force (strong wave),  $M_F$  is the moment of the tension

in the mooring lines, and  $M_B$  is the moment of the buoyant fore (restoring couple). The absolute value of the disturbing moment guarantees the flexibility of the stability relation in the two senses of rotation; that is the couple produced by the weight must also be in opposite sense of the disturbing moment to be capable to right the structure to its initial position.

Hence, the stability constraint can be expressed under the form of an inequality for an optimization problem:

$$C_2(\rho) = -W \left( \frac{x_1^2}{12L} - y_g + \frac{L}{2} \right) \sin\theta - F \cos(\alpha - \theta)x_g + F \sin(\alpha - \theta)y_g$$

$$+ \int_{-L+y_g}^0 (a \cosh k(z+d-y_g) + b \cosh 2k(z+d-y_g) + f)z dz$$

$$+ \int_0^{h-y_g} (a \cosh k(z+d-y_g) + b \cosh 2k(z+d+y_g) + f)z dz \leq 0$$

$h$  is the height of the breakwater portion above the still water,  $\alpha$  being the angle formed by the mooring lines and the vertical  $\alpha=20^\circ$  (chosen to be a small angle in order to heighten the vertical component of the cable tension, which helps the buoyancy couple in restoring the breakwater to its initial position), and  $\theta$  is the angle of disturbance (heeled angle); in fact it is fixed by the designer, and since the breakwater must be very rigid and stable in order to protect the ports from waves, it is taken  $1.2^\circ$  (slope of 2%).

### Structural Constraints

This constraint constitutes a pure structural analysis of the floating breakwater, where a comprehensive numerical analysis is requested in order to determine the mechanical stresses that must be restricted to certain limits. It can be summarized by maximizing a desired property of the structure, mechanical stresses (stiffness), having a given amount of mass distribution. Then, the floating breakwater is modelled using the finite element method fixed on two simple supports at its bottom. It is well known that the concrete have different compression and traction limits due to its nature, and so the well known formula of Von Mises for elastic materials cannot be used. A special criterion, named the Parabolic Criteria, (Garrigues.J, 2001) mainly used for concrete is introduced in terms of the principal stresses of the breakwater and the limit stresses for the material, and is written directly in the form of optimization constraint:

$$C_3(\rho) = (\sigma_1 - \sigma_2)^2 - (\sigma_t + \sigma_c)(\sigma_1 + \sigma_2) - \sigma_t \sigma_c \leq 0 \quad (6)$$

where  $\sigma_1, \sigma_2$  represent the principal stresses of the structure and  $\sigma_t, \sigma_c$  represent the limiting stresses for the material constituting the studied structure. This constraint as the others must be computed in each iteration, which yields to solve the FEM problem in each iteration and for each new defined topology in order to define the principal stresses. In fact, this is the heaviest constraint between the others, where in each iteration a new geometry is defined, meshed, and then passed to the pdeool to be solved.

### FINITE ELEMNT METHOD

The numerical analysis of the mechanical behaviour of this floating breakwater is based on the finite element method (FEM) using the software Matlab. In fact, Matlab solve the problems of (FEM) under the partial differential equations toolbox (PDE Tool), where the mechanical problem is assimilated to an elliptic equation under the form:  $-\text{div}(c \times \text{grad}(u)) + a \times u = f$  in  $\Omega$ , where  $\Omega$  is a bounded domain in the plane,  $u$  is the solution vector,  $c, a, f$  are complex functions

defined on  $\Omega$ . In structural mechanics the main problem is concentrated in solving the equilibrium equation  $\text{div} \vec{\sigma} + \vec{f}_v = \vec{0}$  in a determined structural domain exposed to different boundary loadings (forces and displacements). To solve this classical equilibrium equation under the elliptic family of equations, the elliptic coefficients  $u, c, a, f$  are defined in terms of their equivalence substitutes in a mechanical problem. The  $u$  represents the nodal displacement vector in the two directions,  $a$  equals to zero,  $f$  represents the volume forces or simply the weight ( $-\rho_m \times g$ ), and  $c$  stands for the matrix deduced from the stress-strain relation, assuming isotropic and isothermal conditions.

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{pmatrix} = \frac{E}{1-\nu^2} \begin{pmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & (1-\nu)/2 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{pmatrix}$$

where  $\sigma_x$  and  $\sigma_y$  are the stresses in the  $x$  and  $y$  directions, and  $\tau_{xy}$  is the shear stress. The material properties are expressed as a combination of  $E$ , the elastic modulus or Young's modulus, and  $\nu$  Poisson's ratio.

The basic finite element procedure starts by describing the geometry of the domain  $\Omega$  and the boundary conditions. The boundary conditions specify a combination between  $u$  and its normal derivative on the boundary, and are defined either under the Dirichlet form (defining displacement) or under the Neumann form (defining forces). Second, a triangular mesh is built up on the domain  $\Omega$ ; and finally the structure is discretized into many subregions and for each subregion the displacement field is written in terms of nodal values. The total potential energy is then minimized with respect to the nodal values to give the equilibrium relation

$$\{F\} = [k] \times \{u\}$$

where  $\{u\}$  is the vector of nodal displacements,  $\{F\}$  is the vector of element nodal forces, and  $[k]$  is the element stiffness matrix.

Once the displacement vector  $u$  is computed, it is easy to move deeper and calculate the mechanical stresses and finally the principal stresses, where these latter stresses are the one substituted in the structural constraint expression.

### APPLICATION and RESULTS

Without any further doubt, the proposed method will produce a floating breakwater with a new mass distribution providing an idea of an efficient breakwater. In this section a numerical application is developed and results are obtained based on the following numerical setup for the waves and the dimensions of the breakwater (the width of the breakwater,  $x_1$ , is chosen due to structural demands and concluded from Elchahal,- July 2006):

$$\text{wave} \begin{cases} L = 120m & T = 9 \text{ sec} \\ H = 2m & t = 0 \\ d = 40m & r = 0.8 \end{cases} \quad \text{Breakwater} \begin{cases} \rho_m = 2300 \text{ Kg} / m^3 \\ x_1 = 8m \\ D = 6m \end{cases}$$

The main properties of the GA are as follows:

$$\begin{cases} \text{Individual length} = 309, & \rho = (\rho_1, \dots, \rho_{309}) \\ \text{Population type: Bit String} \\ \text{Crossover fraction: 0.5} \\ \text{Mutation: Adaptive feasible} \\ \text{Crossover: Scattered} \end{cases}$$

By this formulation we can reproduce half of the individuals by mutation and half by scattering in each population, off course other than the elite count. This constitutes a reasonable setup in the GA since scattering or mutation alone is ineffective at all; and by specifying the

population type to Bit String, each density element will conserve its binary representation during mutation. Once again, in optimization problems the initial population plays an important role in drawing a general view for the final solution. In our method, we can control the initial population or solution; that is we define the latter through a program that generates density vectors representing an extracted or void group of triangles that can be accumulated in a void domain. By this way, we avoid falling in trivial solutions when the initial population is representing only arbitrary void triangles. Finally, we obtain the following solution or mass representation for our floating breakwater regarding its conserved external dimensions (rectangle 8\*6)

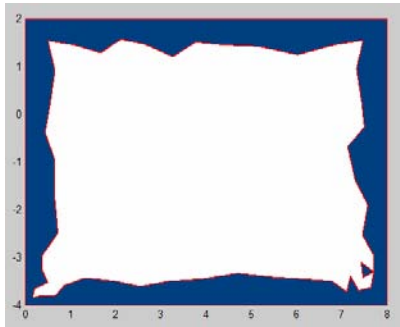


Figure 7 Floating breakwater using topology optimization

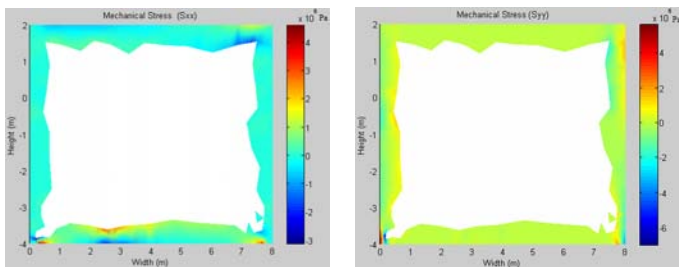


Figure 8 Mechanical stresses  $\sigma_x$  (left) and  $\sigma_y$  (right)

In consequence to our applied method, it is apparent that we ended up with a very logical and accepted solution to our problem considering an overall breakwater problem. In fact, it is not only a problem of volume consuming, but also a structural advantage where the floating breakwater is working approximately in the same stress domain (Fig.8); while in the case of a fixed bottom breakwater (filled material breakwater) the stress domain is largely varying between the points inside the breakwater (Fig.9), Elchahal,- July 2006. This is an additional advantage for the floating breakwater, since the more the inside points are working on closer stresses values the more the extended life of the structure is expected and vice versa. Moreover, we can notice (Fig. 8) the respected limits of the mechanical stresses due to the imposed structural constraints, where the concrete has its traction and compression limits as follows:  $\sigma_t = 4MPa$  ,  $\sigma_c = -40MPa$  .

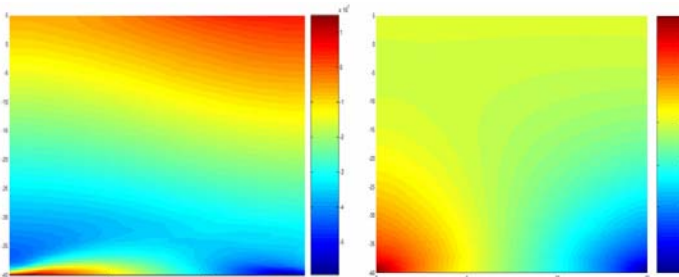


Figure 9 Mechanical stresses for a fixed bottom breakwater  $\sigma_x$  (left) and  $\sigma_y$  (right)

## CONCLUSION

Further to the previous discussions, the robustness of the GA clearly appears in our problem beginning with treating a large number of variables and irregular nonlinear constraints, passing by the evolution of population in each generation, and finally arriving at a logical non-trivial solution. This work constitutes a comprehensive study on floating breakwaters, since it commenced with modelling sea waves and determining its induced pressures on structures. Second, it considered the physical constraints such as floating and stability, where this latter constraint is mainly derived from the sea wave's pressure; also the essential constraint in structures design represented by limitations of the mechanical stresses in not omitted. Then, all the preceding constraints are accumulated in a topology optimization problem using GA, where they are expressed in terms of a density vector characterizing the small decomposed triangles in the whole geometry domain.

## FUTURE RESEARCH DIRECTION

From the synthesis of the optimization procedure presented here and the related discussions, it is evident that a structural technique for floating breakwater design is elucidated. This constitutes an approach to the wave structure interaction problem based on hydrodynamic pressure modelling (assuming that all the waves are reflected by the breakwater with no wave transmission under it due to the fixed sea wall concept) in order to implement the structural optimization of the floating breakwater. For future research, some specific parameters have not been adequately investigated; such as the wave radiation problem and the dynamic behaviour of the floating breakwater. Hence, a comparative example with a floating breakwater basically designed on the basics of wave radiation and transmission is demanded to qualify our presented obtained results. In other words, this work is mainly concentrated on design optimization with structural constraints; where future work must be oriented towards the conventional design with wave transmission constraints. Then, it is important to conclude which of these constraints (structural or wave transmission) is playing an important role in the design procedure and dominating over all the others. Finally, it is important to note that the obtained results, with such complicated section and with sharp edges, is certainly not so good than a smooth section. But, the importance of this method of density distribution lies in two important actualities. The first being a step on the road in topology optimization of marine structures and more particularly opens a gap for its applications to complicated external shapes used in this domain; where it will be very helpful in drawing an initial structural design which will be latterly followed by a shape optimization to smooth such sharp edges. The second fact is that any success of this method on practical applications will open up a new methodology to be benefited from it in inclusion of different materials inside the structure. For example, new applications can be implemented for floating breakwaters made up from concrete and polystyrene by detecting the distribution of these layers inside it based on the density distribution methodology.

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