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WAVE INTERACTION WITH FIXED AND FLOATING VERTICAL BREAKWATER BASED ON ANALYTICAL MODELLING

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ABSTRACT
Nowadays, modelling and computer simulation are used in structure design to reduce the use of experimental investigations and laboratory tests required in a real layout structure. Using the potential-flow theory, the hydrodynamic pressure deduced from sea wave's propagation has been computed based on the non linear theory of Stocks. The extension to the vertical breakwater (fixed bottom and floating breakwater) behaviour due to such excitations is indicated in this paper. Analytical expressions for functional performance variables of wave reflection by vertical breakwater, and stress analysis inside the breakwater have been obtained. These expressions have been numerically verified to demonstrate the capability of this analytical model.

Keywords: Analytical modelling, vertical breakwater, floating breakwater, wave structure interaction, stress analysis, Matlab.

INTRODUCTION
Coastal structures are designed mainly to provide protection by reflection and/or dissipation of wave energy. Rubble-mound and fixed bottom vertical breakwaters have been extensively used for sheltering harbours [1], where most of these caisson-type structures are monolithic structures with impermeable vertical walls. Nevertheless, at many locations, the site specific parameters as deep water or poor bottom conditions as well as environmental requirements including the phenomena of intense shore erosion, water quality and aesthetic considerations, advocate for the application of such floating structures.

Floating breakwaters present an alternative solution to conventional fixed breakwaters, but it cannot be effectively used except in coastal regions with mild wave environment conditions. Such breakwaters have many advantages compared to the fixed conventional ones. The most important being the cost effectiveness, flexibility of future extensions, mobility, and reallocation ability. Research engineers and scientists have realized the potential for floating breakwaters in certain areas, and research interest has been directed towards this subject during the last decades. As a result, many types of floating breakwaters have been developed, as described by McCartney (1985), however, the most commonly used are the rectangular pontoon-type breakwaters, which are moored to the sea bottom with cables or chains. Moreover, many studies have been produced on floating breakwaters (Twu and Lee, 1983; Johansson 1989; Murali and Mani, 1997; etc.), mainly concerning the wave protection improvement by different types of floating structures. Other studies have been directed towards the mooring forces and motion responses to understand the behaviour of the floating breakwaters due to sea waves (Williams and Abul-Azm,1997; Sannasiraj, 1998; and Lee and Cho, 2003). But still all these proceeding studies are performed for special type of floating breakwaters that can be used only in mild environment or in places that require a lower level of stability; while this paper seeks to develop a floating breakwater until that would have the capability of withstanding more severe wave loading conditions such that these structures will become a viable alternative to conventional breakwaters for moderately exposed locations. This requires a comprehensive structural analysis study to develop an arguable comparison between the two types (fixed bottom and floating) to prove the capability of a well designed floating breakwater.

Although, the protection of marine structures has been extensively studied in recent years, understanding of their interaction with waves, marine structures and the seabed is far from complete (D.Jeng, 2005). Damage of marine structures still occurs from time to time, with two general failure modes evident. The first mode is that of structural failure, caused by wave forces acting on and damaging the structure itself. The second mode, which has attracted many of the scientists (Biot-1941; Jeng 1997; Mitzutani 1998), is that of foundation failure caused by liquefaction or erosion of the seabed in the vicinity of the structure, resulting in collapse of the structure (case of fixed bottom breakwaters only), where our work is mainly directed towards the structure failure due to the lack of knowledge in this domain. Moreover, the physical understanding and computation of wave–structure interaction, one of the most important hydrodynamic processes in both coastal and offshore engineering, are crucial to assess wave impacts on structures as well as structural responses to wave attacks. Traditionally, the estimation of wave loads on a structure is often done by either empirical approach (ex: Morison equation Sainflou, Hiroi, Goda, Svendson…) or a computational approach (G. Gruhan, 2005). The empirical formulas are simple but crude and will not be able to provide detailed and accurate information about pressure distribution on a structure. The computational approach can be further divided into two types: the Laplace equation solver for potential flows (D.Jeng, 2005) and the Navier–Stokes Equations (NSE) solver for viscous flows, where the latter is used for simulation of wave–structure interaction during which both vortices and turbulence may be present, where solving the Laplace equation by imposing the boundary conditions constitutes the wave modelling part in this study.

Finally, the methodology followed in this paper is first identified by an analytical modelling of waves and their induced pressures exerted on vertical breakwaters and finally modelling the behaviour of the breakwater (fixed and floating) due to wave-structure interaction; where it is obviously to mention that the same wave characteristics are applied for both cases. It is interesting to consider the case of a vertical breakwater appearing in ports' constructions far from the shore, at a constant depth, and at a fixed point. Then, the problems of wave's propagation over a varying bathymetry and shallow water consequences are eliminated.

WAVE MODELLING

A cartesian coordinate system *Oxyz* is employed, where *Oxy* coincide with plane of the free surface at rest, *Oz* directed positive upwards, and *Ox* directed positive in the direction of propagation of the waves. The incident wave propagates in a straight line in the direction defined by the angle γ, formed with the *Ox* axe. In this study, it is supposed that the waves can strike the breakwater in a perpendicular direction to obtain the maximum pressure applied by the waves on the breakwater, in order to study the dangerous case in the construction of a breakwater. Then, the angle is taken as $\gamma = 0$ (incident wave normal to the breakwater) and the movement is reduced to two dimensions figure 1.

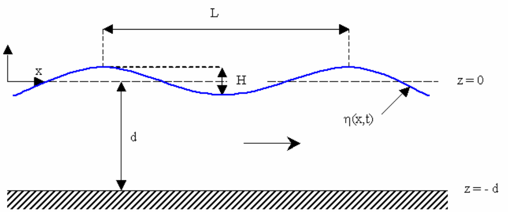


Figure 1 Wave notations

The fluid motion is defined as follows: Let t denote time, x and z the horizontal and vertical coordinates, respectively, and η the free-surface elevation above the still water level. The high values of the density and sound velocity in water render the compressibility effects negligible in sea water, so it is considered incompressible. The fluid is considered also irrotational. Then, the fluid motion can be described by a velocity potential, Φ, related to the velocity $\vec{U}(u,w)$.

$\overrightarrow{rot}(\vec{U}) = \vec{0} \Rightarrow \vec{U} = \overrightarrow{grad}(\Phi)$, where $u = \frac{\partial\Phi}{\partial x}$ and $w = \frac{\partial\Phi}{\partial z}$.

Once the parameters characterizing the sea waves are known (Length of wave L, Period T, Height H), a model is needed to study the waves' propagations and transforms their evolution into loads on the breakwater. It is a strict study based on the fundamental physical principles of the conservation of momentum and mass. The combination of the equation of momentum conservation and that of mass, yields to the well known equation, Bernoulli-Lagrange, which constitutes the essential equation to determine the field of wave's pressure.

$$\frac{\partial\Phi}{\partial t} + \frac{1}{2}(grad\Phi)^2 + \frac{P(x,z,t)}{\rho} + gz = Q(t)$$

In general, the study of marine structures' behaviours due to waves' propagations is mostly made as part of a linear theory (B. Molin, 2004), where the interest in this paper is to orient the work towards the non linear approximation (Stokes 2nd order expansion), which yields to a clarified view of the efforts in an enlarged domain of frequencies and moreover the linear wave theory is not expected to have outstanding agreement with the real and experimental data.
It is clear that if Φ is known throughout the fluid, the physical quantities (pressure and velocity) can be obtained from Bernoulli's equation. The boundary value problem is then defined as follows:

$\nabla^2\Phi = \Delta\Phi = 0$ Laplace equation in the fluid domain;

$\left(\frac{\partial\Phi}{\partial z}\right)_{z=-d} = 0$ Condition at the sea floor;

$\left(\frac{\partial\Phi}{\partial n}\right)_{x=0} = 0$ Kinematic condition at the solid boundary;

$\left(\frac{\partial\eta}{\partial t} + \frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x} - \frac{\partial\Phi}{\partial z}\right)_{z=\eta} = 0$ Kinematic condition at the free surface;

$\left(\frac{\partial\Phi}{\partial t} + \frac{1}{2}\left(\left(\frac{\partial\Phi}{\partial x}\right)^2 + \left(\frac{\partial\Phi}{\partial z}\right)^2\right) + g\eta\right)_{z=\eta} = Q(t)$ Dynamic equation at the free surface;

The equation of Laplace expresses the mass conservation; the sea bottom condition expresses the impermiability of the sea bed where the normal component of the velocity is zero; the kinematic condition at the solid boundary (breakwater, $x = 0$), expresses the static condition of the breakwater (wave reflection) where $\vec{n}$ is the outward normal direction of the solid boundary; the kinematic condition on surface, $z = \eta$, expresses that a fluid particle at the surface should remain there at all times, while the dynamic condition expresses that the pressure on the free surface is zero. The used method for the nonlinear theory (Stockes 2nd order expansion), called perturbation method, consists of developing the different variables into power series depending on a parameter $\varepsilon = \frac{H}{L}$, where the linear theory constitutes the first order yielding exact solutions only for waves with infinitesimal amplitudes.

$$\Phi = \varepsilon\Phi_1 + \varepsilon^2\Phi_2 + \varepsilon^3\Phi_3 + ... + \varepsilon^n\Phi_n$$

By considering the amplitudes of the oscillations of the free surface to be small, the terms are then evaluated on the free surface depending on $\eta(x,t)$ due to Taylor series.

$$\Phi(x,\eta)=\Phi(x,0)+\eta\left(\frac{\partial\Phi}{\partial z}\right)_{z=0}+\ldots\ldots\ldots+\frac{\eta^n}{n}\left(\frac{\partial^n\Phi}{\partial z_n}\right)_{z=0}$$

The developments are limited to the second order of the camber ε so: $\Phi=\varepsilon\Phi_1+\varepsilon^2\Phi_2$ and $\eta=\varepsilon\eta_1+\varepsilon^2\eta_2$. It is convenient to determine $\Phi_2(x,z,t)$ and $\eta_2(x,t)$ knowing Φ_1 and η_1 (linear case), Then the boundary conditions for the free surface for $z=\eta(x,t)$, are transformed into perturbation series. Solving for the 1st order expansion (linear theory)

$$\Phi_1=\mathrm{Re}\left\{-i\frac{Hg}{2\omega}\frac{ch[k(z+d)]}{ch(kd)}\exp i(kx-\omega t)\right\}\ ,\quad \eta_1=\mathrm{Re}\left\{\frac{H}{2}\exp i(kx-\omega t)\right\}$$

(Where $k=2\pi/L$ designates the wave number and ω the frequency). The nonlinear approximation is achieved by substituting for the first order in the perturbation series:

$$\Phi(x,z,t)=\mathrm{Re}\left\{-i\frac{Hg}{2\omega}\frac{ch[k(z+d)]}{ch(kd)}\exp i(kx-\omega t)\right\}-\frac{\pi gH^2}{4L}\frac{t}{sh(2kd)}$$
$$+\mathrm{Re}\left\{-i\frac{3}{8}\left(\frac{H}{2}\right)^2\frac{gk}{\omega}\frac{ch[2k(z+d)]}{sk^3(kd)ch(kd)}\exp 2i(kx-\omega t)\right\}$$

This expression of velocity potential describes the physical properties of the waves in the absence of any structure, but the reflection phenomenon must be taken into consideration during the collision of the waves by the breakwater. Then, a reflected wave identical to the incident one is created but in the opposite sense. $\Phi_r(x,z,t)=r\times\Phi_i(-x,z,t)$

Where r designates the reflection coefficient (coefficient of amplitude reduction), the superposition of the incident and reflected velocity potentials creates a global wave system (Y. Goda, 1985) whose velocity potential is defined as: $\Phi_T=\Phi_i+\Phi_r$.

Moreover, the extremity of the breakwater involves the diffraction of the waves and hence concentric circles are formed around its extremity. Considering a semi-infinite breakwater, eliminates this phenomenon and keeps the problem in the domain of wave reflection only; where the global potential velocity describing the problem is maintained as expressed above. The substitution of this value for the velocity potential (Φ_T) in the Bernoulli-Lagrange equation implies the expression of the pressure distribution (pressure at any point in the fluid domain.) in the case of wave-breakwater interaction, where all the waves are reflected by the breakwater (no diffraction or transmission).

$$P(x,z,t)=-\rho gz+\mathrm{Re}\left\{\begin{array}{l}\frac{1}{2}\rho gH\frac{ch[k(z+d)]}{ch(kd)}\\ \left[\exp i(kx-\omega t)+r\exp i(-kx-\omega t+\beta)\right]\end{array}\right\}$$
$$+\mathrm{Re}\left\{\begin{array}{l}\frac{3}{4}\rho gH\frac{\pi H}{L}\frac{1}{sh(2kd)}\left[\frac{ch2k(z+d)}{sh^2kd}-\frac{1}{3}\right]\\ \left[\exp 2i(kx-\omega t)+\left(r^2+r\right)\exp 2i(-kx-\omega t+\beta)\right]\end{array}\right\}$$
$$+\mathrm{Re}\left\{r\rho H^2\omega^2\exp i(-2\omega t+\beta)\right\}-\frac{1}{4}\rho gH\frac{\pi H}{L}\frac{(r+1)}{sh(2kd)}\left[ch2k(z+d)-1\right]$$

FIXED BREAKWATER MODELLING

There is no experience at hand to premeditate the destruction failures or damage of breakwater at design stage itself, and this return to the lack of detailed structural studies. The analytical method proposed in this paper for the analysis of structural construction of conventional breakwaters is based on predicting the hydrodynamic forces (induced from the waves), and then analyzing the stress repartition in the breakwater by considering the hydrodynamic forces and the inertia forces due to rigid-body motions as external forces. This study is assimilated to a mechanical problem in a continuous medium, which consists to determine analytically the stress tensor $\sigma(M)$ on each point M in the solid of the studied structure (S. Timoshenko, 1961). The general method used in elasticity theory to determine the stress distribution in the interior of an elastic body, is based on proposing the form of the stress tensor that must satisfy the equilibrium equations, all the boundary conditions, and the compatibility equations. The disadvantage of this method is that it needs a lot of function trials (especially with such non uniform equation of hydrodynamic pressure), since there many functions that satisfy the above conditions but not all of them describe the real state of the problem. For the simplicity of calculations, it is recommended to divide the problem over two parts: the hydrodynamic forces developed from the waves' propagations and the hydrostatic forces developed from water depth.

Hydrodynamic pressure

The exerted pressure by waves on the vertical breakwater is deduced from the computed fluid problem in the first section. This hydrodynamic pressure has a complicated expression different from the hydrostatic one that is linear, its repartition over the breakwater has a curved shape (obtained using Matlab); where its maximum is around the still water level and it decreases to zero at the top of the breakwater (with the wave height) and also decreases with water depth (figure 2). Fixing $x=0$ (exterior breakwater surface), and the phase angle $\beta=0$ (vertical impermeable wall), the pressure distribution over the vertical breakwater is obtained.

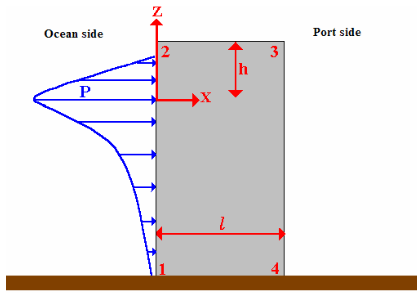


Figure 2 Hydrodynamic pressure distribution over the breakwater

This hydrodynamic pressure is acting on the exterior surface of the breakwater due to the assumption that all the waves propagating from the ocean side are totally reflected outside the port (no transmission); and hence it can be simply deduced that there are no dynamic pressure acting on the interior surface of the breakwater due to the absence of waves' propagations inside the port. It can be written as follows:

$$P=a\cosh k(z+d)+b\cosh 2k(z+d)+f$$

$$a=\frac{\rho gH}{2}\frac{(r+1)}{chkd}\cos(\omega t)\ ,\quad b=\frac{\rho g\pi H^2}{4Lsh2kd}\left[\frac{(3r^2+3r+3)\cos(2\omega t)}{sh^2kd}-r-1\right]$$

$$f=\frac{\rho g\pi H^2}{4Lsh2kd}\left[\left(-r^2-r-1\right)\cos(2\omega t)+r+1\right]+\rho H^2\omega^2 r\cos(2\omega t)$$

It is reduced to an equation with hyperbolic functions of z (altitude), where the other variables independent of the altitude are collected together in the terms $a, b,$ and f. The hydrodynamic stress's form is chosen as:

$$\sigma_{xd} = A_1 z \cosh k(z+d) + B_1 \sinh k(x-l) + C_1 \cosh 2k(z+d)$$

$$\sigma_{zd} = A_2 z \cosh k(z+d) + B_2 (z-h)^3 \sinh k(x-l) + C_2 x^3 \cosh 2k(z+d) + D_2 (x-l)^3 \cosh k(z+d)$$

$$\tau_{xzd} = A_3 z \cosh k(z+d) + B_3 z^2 \cosh k(x-l) + C_3 x^2 \cosh 2k(z+d) + D_3 (x-l)^2 \cosh k(z+d)$$

where terms $A_1, B_1, C_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3$ are to be determined when applying the equilibrium, boundary, and compatibility equations.

a-Equilibrium conditions:

$$\overrightarrow{div\,\sigma} + \vec{f_v} = \vec{0} \quad , \quad \begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \sigma_z}{\partial z} = \rho_b g \end{cases}$$

b-Boundary conditions: The condition for the ocean side (1-2) is : $\overline{\overline{\sigma}}.\vec{n} = \vec{\sigma_f}$, where $\vec{n}$ is the normal to the side (1-2), which is equal to (-1,0).

The constraint vector $\overline{\overline{\sigma}}.\vec{n}$ is equal to the surface force $\vec{P}$.

For the port side, there is absence of any type of dynamic pressure, then on $x=l$: $\overline{\overline{\sigma}}.\vec{n} = \vec{0}$,and also for the upper side of the breakwater, no forces are exerted, then on $z=h$: $\overline{\overline{\sigma}}.\vec{n} = \vec{0}$.

Finally, it ends up with 8 equations.

c-Compatibility equation: The problem is statically undetermined by the above conditions alone, and to solve such problem elastic deformations must be introduced. The mathematical expression of the compatibility equation of the stress distribution with the existence of the continuous functions U and V defines the deformation state. This differential equation, compatibility equation, assures the existence of the deformation functions and satisfies the boundary conditions also. (the 6 compatibility equations are reduced to one equation in the case of 2D)

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z}, \text{ where}$$

$$\varepsilon_x = \frac{\partial U}{\partial x} = \frac{1}{E}(\sigma_x - \nu\sigma_z), \quad \varepsilon_z = \frac{\partial V}{\partial z} = \frac{1}{E}(\sigma_z - \nu\sigma_x), \quad \gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial V}{\partial x}$$

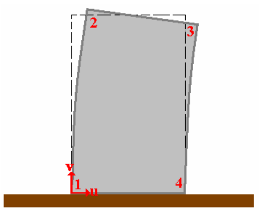


Figure 3 Fixed support stability conditions

The integration of the above expressions yields to determine the deformations' functions for each point in the solid; where q and r (integration constants) are two functions chosen to satisfy the sufficient conditions to identify the problem:

$q(z) = m \sinh k(z+d)$, m is a variable to be determined

$r(x) = 0$, chosen equal to zero for the lack of any additional boundary condition. The boundary conditions applied at the bottom of the breakwater are related to the state of breakwater foundation, figure 3.

Fixed supported bottom $\begin{Bmatrix} U(z=-d)=0 \\ V(z=-d)=0 \\ \gamma(z=-d)=0 \end{Bmatrix}$ whatever the value of x. The problem arrives to 11 variables with 11 equations which describe the dynamical physical state of the wave-breakwater interaction. Solving for these variables, it is ended with the hydrodynamic stress tensor.

Hydrostatic pressure

The hydrostatic pressure exerted on the breakwater due to water depth has a linear form which simplifies the problem (figure 4). The difference between the water levels on each side of the breakwater is due to the elevation of the water level from the ocean side due to waves' propagations (h≈1.25 wave height). The stress tensor for the hydrostatic problem is supposed as follows:

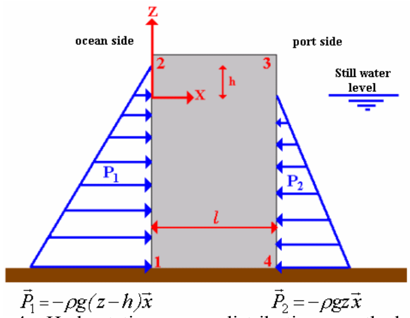


Figure 4 Hydrostatic pressure distribution over the breakwater

$$\sigma_{xs} = A_1 x^2 + B_1 z + C_1$$

$$\sigma_{zs} = A_2 xz(x-l) + B_2(z-h) + C_2 x(x-l) + D_2$$

$$\tau_{xzs} = A_3 xz(x-l) + B_3 x(z-h)^2 + C_3 x^2 + D_3$$

Repeating the same procedure for the hydrostatic pressure problem, 11 variables with 11 equations, it is ended by the hydrostatic stress tensor definition. The global problem defining the real existing case is attained by combining the hydrostatic and hydrodynamic problem. This leads to an analytical addition for the resultant stresses defining the whole problem.

$$\sigma_x = \sigma_{xs} + \sigma_{xd}, \quad \sigma_z = \sigma_{zs} + \sigma_{zd}, \quad \tau_{xz} = \tau_{xzs} + \tau_{xzd}$$

The stress tensor is then finally defined by extremely complicated and long equations (no space to list them) in function of the wave properties and the coordinates of any point on or inside the contour of the breakwater. In fact, it is very important to end up with such equations capable to define the stresses at any point of this breakwater, where it is deduced from analytical models.

Stability against overturning

After defining all the exerted forces on the breakwater and the stresses' distributions inside it, it is important to introduce stability condition to maintain equilibrium despite all these exterior forces, where the weight of the breakwater must remain sufficiently stable to resist rotation under the impact of waves. The verification equation for structure stability, is the well known condition for overturning around the lower inner edge of the breakwater. $M_w - M_d - M_s \geq 0$; where M_w is the moment of the weight, M_d is the moment of the hydrodynamic pressure applied by the waves, and M_s is the moment of the hydrostatic pressure acting on the whole structure $(P_1 - P_2)$. From this stability condition, the width of the vertical breakwater can be simply deduced to stabilize it against overturning from the following expression: (ρ_m designates the density of the inside material)

$$l^2 \geq \frac{2}{\rho_m (d+h)} \int_0^{d+h} (a \cosh k(z+d) + b \cosh 2k(z+d) + f) z dz$$
$$+ \frac{2}{\rho_m (d+h)} \left[\int_d^{d+h} (\rho(-z+d+h) z dz + \int_0^d \rho h z dz \right]$$

Verification of results

The success of the theoretical and analytical formulation of the wave-vertical breakwater interaction relies on the accuracy and accordance of these results with the numerical ones. The setup for an analytical calculation is as flows (choosing the parameters of a strong wave): wave properties [L=140 m, T=9 sec, d=40 m, H=4 m, r=0.8, t=0, sea water density=1025 kg/m^3] and breakwater properties [l=15 m (satisfying stability condition), concrete density=2300 kg/m^3, υ=0.2, E= 20x10^9 N/m^2], where all the calculated stress equations are programmed in MATLAB in order to benefit from its high quality in treating symbolic equations and drawing their evaluated contours. To verify our results, a comparison made with numerical results based on the finite element theory also using MATLAB in its partial differential equation toolbox, PDE tool. (see figure 5)

Finally, an analytical model based on the potential flow theory for the wave propagation and on the mechanics of continuous medium for the stress distribution inside the structure, has been set up to study the wave-vertical breakwater interaction in this paper. By comparing with numerical data (pdetool MATLAB), this analytical model has been proven to be reliable to simulate the waves propagations, their induced pressures on the breakwater, and the structure behaviour due to the applied forces. Regardless to the difference in the stress distribution at the breakwater fixed support, the results of the analytical model are in good agreement with the numerical data, and there are some important remarks to be concluded from these interesting results:

1- The maximum stresses' values result from the huge weight of the structure, from the hydrostatic pressure due to the water depth, and not from the hydrodynamic pressure induced form the waves' propagations, and hence the breakwater that is mainly designed to protect the ports from the waves' propagations is submitting induced pressure out of its objective that is playing an important role in characterising its design and overwhelming over all other constraints. For example, the maximum hydrodynamic pressure obtained from the above wave properties is 0.6 x10^5 Pa where the hydrostatic one reaches 4.5x10^5 Pa at the bottom of the breakwater (see σ_x) and this results correspondingly in very high bending and reaching 4 and -6 MPa (see σ_z and τ_{xz}). These high values for the bending stresses can probably cause the total destruction of the whole breakwater in case of strong waves due to the great traction efforts at the bottom.

2-It is clearly observed that the difference in the stress contours distribution, near the fixed support between the two cases, returns to the two different methods in calculating the stress; where the support conditions appear only in the stress σ_z for the case of analytical modelling. On the other side, the numerical modelling takes into considerations the support conditions in calculating all the stresses by the method of finite element.

3- The study must be oriented towards the floating breakwater instead of the fixed one, to suppress the great height of the breakwater causing the enormous hydrostatic pressure and the enormous weight of the structure; moreover to suppress the second important mode of failure summarised in the erosion of the seabed in the vicinity of the structure.

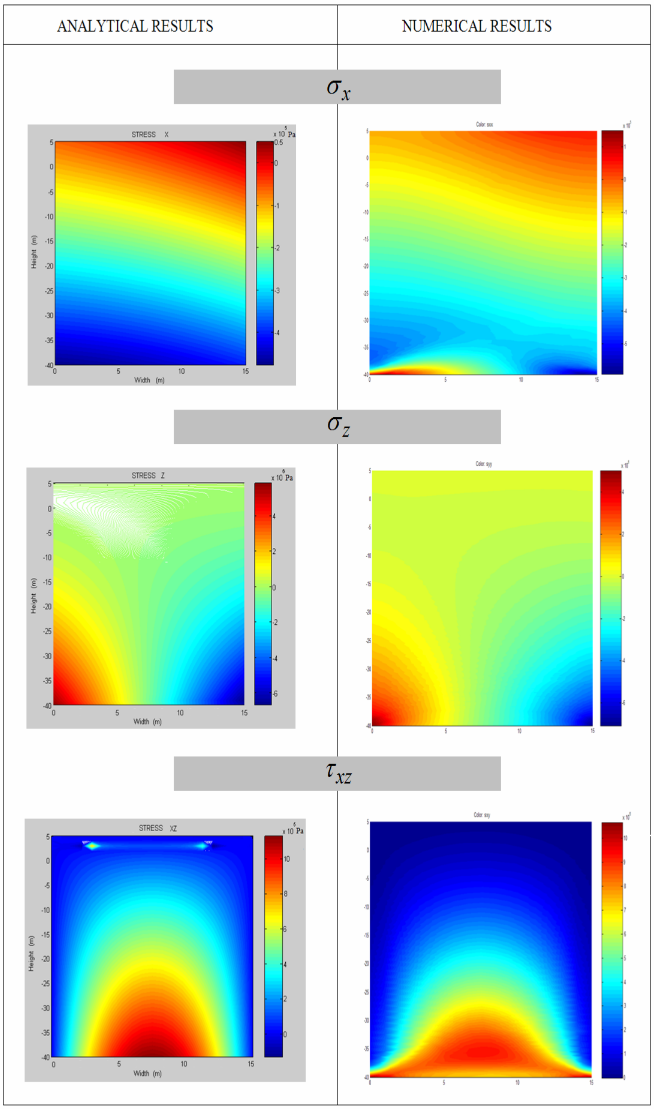


Figure 5 Analytical-numerical results for a fixed breakwater

FLOATING BREAKWATER MODELLING

A moored floating breakwater should be properly designed in order to ensure: (a) effective reduction of the transmitted energy, hence adequate protection of the area behind the floating system, (b) non-failure of the floating breakwater itself and (c) non-failure of the mooring lines. The satisfaction of these 3 requirements represents the overall desired performance of the floating breakwater. The non-failure of the mooring lines has been widely studied and discussed, so the efforts in this paper are directed towards the first two issues. The reduction of the transmitted energy is achieved by the floating breakwater itself due to a considerable depth and by the fixed seawall concept under the breakwater for the rest underwater region. This fixed seawall works by stimulating the inertia of the mass of water between the lower side of the caisson and the sea bed, which then reacts like a real wall against which the surge is reflected. By adjusting the width and submersion of the caisson, its characteristics can be adapted to the strength of the surge it has to attenuate. Moreover, for a breakwater to float it is obviously designed with a hollow form to reduce the total weight of the structure; where such form complicates the problem and implicates more constraints to be considered during the design.

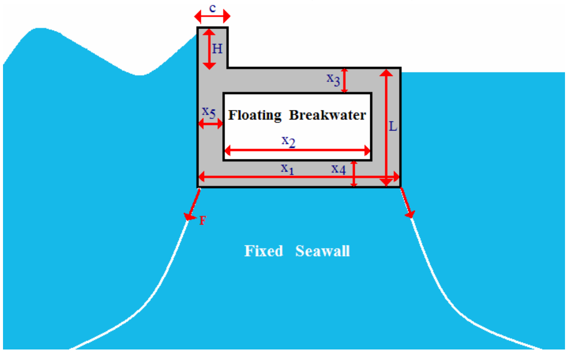


Figure 6 Characteristics of floating breakwater

An additional rectangular wall can be used to protect the sheltered regions from high waves; where it is sufficient to place it only from the ocean side since it has non sense to construct a rectangular breakwater with its height over the free surface level equals to a strong wave height. Then, it can be simply deduced that a floating breakwater can be assimilated to two parts: the main rectangular body possessing sufficient dimensions considering the fixed seawall concept, and a second part formed by a small rectangular wall fixed on the ocean side of the breakwater to attenuate the high waves. The dimensions of the second part are easily determined, where its height is equal to the wave height H, and its width c is taken to be 0.7 m (J. Larras, 1979).

Improving the performance of floating breakwaters such that they can withstand more severe loading conditions and still provide adequate protection could open up multiple of possible uses and this because the floating breakwater, in contrary to the fixed one (the only parameter to calculate is the width being deduced from the stability condition), has many parameters characterizing its geometry and defining its shape $L, x_1, x_2, x_3, x_4, x_5, F$ (see figure 6). Some of these parameters are related to the same physical constraint where the rest are determined from other independent constraints, and therefore determining its geometrical dimensions cannot be performed as an ordinary calculation problem but it needs an optimisation process in order to compute these parameters taking into consideration their effects on each other. The optimisation problem is assumed to be finite-dimensional constrained minimization problem, which is symbolically expressed as:

Find a design variable vector x;

to minimize the weight function $f_{ob}(x)$

subject to the n constraints $f_i(x) < 0$

The problem is then expressed as an objective function to minimize due to the following constraints:

1-Objective function: The optimal solution is to design a breakwater respecting all the constraints with a minimum volume, hence the objective is to minimize the weight of the breakwater,

$f_{ob}(x_1, x_2, x_3, x_4, x_5, F) = Lx_1 - x_2(L - x_3 - x_4) + Hc$

2-Dynamic pressure constraint: The concept of the fixed seawall permits to determine the height of the breakwater in accordance with low hydrodynamic pressure acting on this seawall. The dynamic wave pressure is mainly concentrated near the free surface and its induced perturbation is low under a certain height (see figure 7); then the height of the breakwater can be limited to where the pressure is approximately unvarying corresponding to an approximate value of $P - 0.05P\max = 0$, where $P\max = P(z = 0)$. Finally, the height can be considered to be $L = 8m$, where this height is indeed satisfactory for a strong wave $(H = 4m)$, where it constitutes about $2H$ (H being the height of the sea wave).

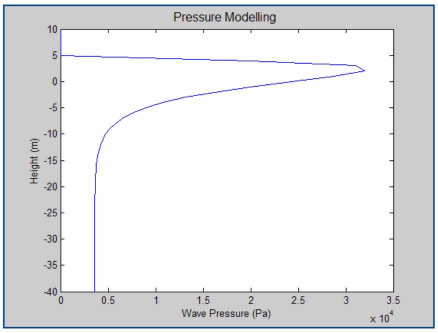


Figure 7 Wave Pressure Modelling

This constraint is independent of the other constraints, and then the height of the breakwater is determined only from it and no need to still consider the height as a variable for the rest of the optimization process.

3- Floating constraint: The forces acting on the floating breakwater are numerous (see figure 8) and of various sources thus they are defined as follows:

P_1=hydrostatic pressure acting on the two sides, $P_1 = \rho gy$

P_2=hydrostatic pressure acting on bottom surface, $P_2 = \rho gL$

P_3=hydrodynamic pressure acting from the ocean side; it is modelled as two triangular forces where the maximum is located at the height z=0 (water free surface) and it is evaluated by substituting the value of z in the equation of hydrodynamic pressure, $P_3\max = a\cosh(kd) + b\cosh(2kd) + f$

The pressures P_1 and P_2 constitute the pressure on the submerged volume of the breakwater, mainly known as Archimedes principle.

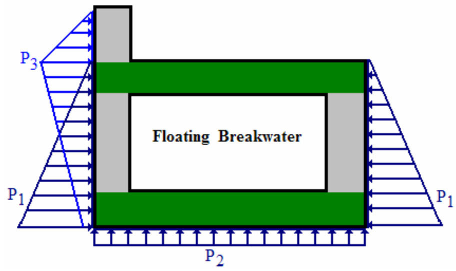


Figure 8 Applied forces on a floating breakwater

In fact, the buoyant force exerted on the breakwater collaborates in an important constraint in the optimisation process, where really this force is the main factor in realizing a floating structure.

The equilibrium equation for floating can be written as:

$-\rho_m(V_m + V_r)g + \rho_e V_T g = 0$,

ρ_m and ρ_e designates the densities of the material and the sea water respectively, V_m designates the volume of the inside material of the whole breakwater without the upper rectangular wall, V_o designates the volume of the hollow part (atmospheric pressure inside), V_r designates the volume of the upper rectangular part, where V_T designates the volume of the submerged part of the breakwater , and then $V_m + V_o = V_T$

A relation between the hollow volume and the submerged volume can be simply deduced: $V_o = \frac{\rho_m - \rho_e}{\rho_m} V_T + V_r$

The floating constraint can be expressed as follows:

$$f(x_1, x_2, x_3, x_4, x_5, F) = x_2(L - x_3 - x_4) - \frac{\rho_m - \rho_e}{\rho_m} L x_1 - V_r$$

But, really the floating constraint yields to a simple relation between the variables that can be used to reduce the number of variables in the optimization process.

$$x_1 = \frac{\rho_m\left[x_2(L - x_3 - x_4) - V_r\right]}{(\rho_m - \rho_e)L} \quad \rho_m$$

4-Stability constraint: Stability is one of the more quantitative aspects of how a floating object behaves in water; thus its analysis is quite different for a floating breakwater than for a fixed one. It is defined as the ability to right itself after being heeled over; this ability is achieved by developing moments that tend to restore the breakwater to its original condition. There are a number of calculated values that together determine the stability of a floating breakwater: 1- Initial horizontal equilibrium, 2- Heeled angle, 3- Tension in mooring lines.

First of all, this floating breakwater has a non-symmetrical shape, so initially (before any disturbance) it is necessary to maintain a horizontal equilibrium position. This is performed by dividing the breakwater into 5 rectangles and calculating the new position of the centre of gravity (see figure 8) in terms of the variables and then aligning it with the centre of buoyancy for the floating breakwater (see figure 9) which lies at the geometric centre of volume of the displaced water ($x_1/2$).

$$\bar{x} = \frac{(L - x_3 - x_4)\left[\frac{(x_1 - x_2 - x_5)^2}{2} + \left(x_1 - \frac{x_5}{2}\right)x_5\right] + \frac{x_1^2}{2}\left[x_4 + x_3\right] + Hc\left[x_1 - \frac{c}{2}\right]}{x_1 x_4 + x_1 x_3 + Hc + (L - x_3 - x_4)(x_1 - x_2)}$$

$$\bar{y} = \frac{\left[(L - x_3 - x_4)\frac{(L - x_3 + x_4)}{2}\right](x_1 - x_2) + x_1\left[\frac{x_4^2}{2} + \left(Lx_3 - \frac{x_3^2}{2}\right)\right] + Hc\left[L + \frac{H}{2}\right]}{x_1 x_4 + x_1 x_3 + Hc + (L - x_3 - x_4)(x_1 - x_2)}$$

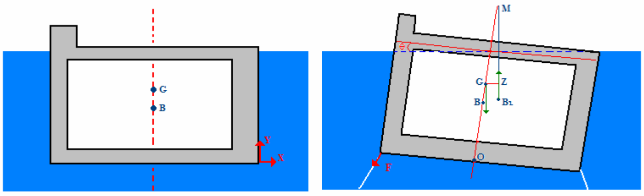


Figure 9 Stability of floating breakwater

When the breakwater is disturbed by a wave, the centre of buoyancy moves from B to B_1 (see figure 9) because the shape of the submerged volume is changed; then the weight and the buoyancy force form a couple capable to restore the breakwater to its original position. Moreover, the distance GM known as the metacentric height illustrates the fundamental law of stability, where it must be always positive to create a restoring couple and maintain stability. The equation of motion can be written as: $\sum M = I\ddot{\theta} \Rightarrow$ at equilibrium $Mp - M_F - M_B = 0$, where Mp is the moment of the disturbing force (wave), M_F is the moment of the tension in the mooring lines, and M_B is the moment of the buoyant fore (restoring couple), the stability constraint can be expressed as

$$f_1(x_2, x_3, x_4, x_5, F) = -Lx_1\rho g\left(\frac{x_1^2}{12L} - \bar{y} + \frac{L}{2}\right)\sin\theta - F\cos(\alpha - \theta)\frac{x_1}{2}$$
$$+ F\sin(\alpha - \theta)\bar{y} + \int_0^{L/2}(a\cosh k(z + d - L + \bar{y}) + b\cosh 2k(z + d - L + \bar{y}) + f)zdz$$
$$- \int_{-L/2}^{0}(a\cosh k(z + d - L + \bar{y}) + b\cosh 2k(z + d - L + \bar{y}) + f)zdz$$

α being the angle formed by the mooring lines and the vertical (α=*20°*), and θ is the angle of disturbance (heeled angle); in fact it is fixed by the designer, and since the breakwater must be very rigid and stable in order to protect the ports from waves, it is taken to be *1.2°*.(slope of 2%)

The second relevant constraint is $\bar{x} = x_1/2$ (horizontal equilibrium condition)

$$f_2(x_2, x_3, x_4, x_5) = -x_1^2(x_4 + x_3) + Hcx_1 + x_1(L - x_3 - x_4)(x_1 - x_2)$$
$$2(L - x_3 - x_4)\left[\frac{(x_1 - x_2 - x_5)^2}{2} + \left(x_1 - \frac{x_5}{2}\right)x_5\right] + 2x_1^2\left[x_4 + x_3\right] + 2Hc\left[x_1 - \frac{c}{2}\right]$$

5-Structural constraints: This constraint constitutes a pure structural analysis of the floating breakwater, where a comprehensive structural study is requested in order to determine the bending moments, stresses, and deflections that must be restricted to certain limits. The floating breakwater is modelled as a frame structure fixed on two simple supports at its bottom, where it can be simply divided into four beams with assimilating the upper rectangular wall as a concentrated force on the upper beam. Each beam is equilibrated by the internal reactions and moments generated from frame division, and hence the equilibrium conditions can be applied for each beam alone to determine the internal efforts and moments yielding to the deflection and stress calculations, (see figure 10) $\Sigma F_x = 0,\quad \Sigma F_y = 0,\quad \Sigma M = 0$. All the forces are distinguished from each other by different colours and are well explained in the figure below.

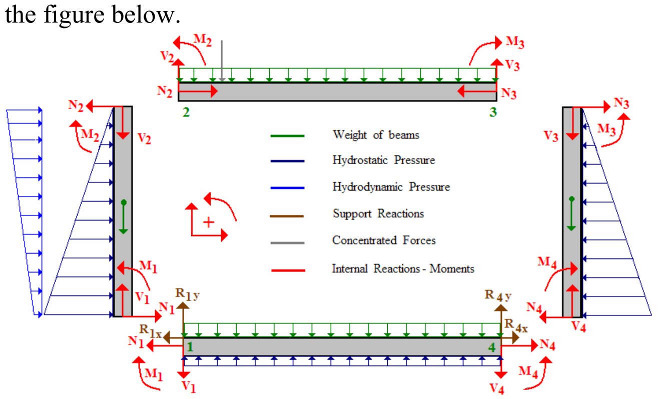


Figure 10 Forces and moments distributions

This constitutes a problem of 12 variables (Ni, Vi, and Mi where $i = 1,2,3,4$) with 12 equations, but in fact there is only 9 effective equations (equilibrium conditions for beam 1-4, 1-2, 2-3) and the last 3 equations (beam 3-4) are linearly dependant and will not help to solve the system of 12 variables. This problem is of the hyper-elastic type, where the number of equations is not sufficient to determine the corresponding variables (J. Roux, 1995), and it is necessary to include three other relations deduced from applying Castigliano's theorem on the fixed nodes (beam 1-4 and 1-2).

$\lambda_i = \dfrac{\partial W}{\partial F_i} = \int M \dfrac{\partial M}{F_i}\dfrac{dx}{EI}$, λ_i being the displacement of the node where the force F_i is applied, and M the distribution of moment along the beam.

The bending moments along each beam is determined by the traditional way of beam theory, $\Sigma M_I = 0$:

Beam 1-2: $0 \le y \le L$

$$M(y) = -M_1 - N_1 y - \frac{(a\cosh(kd) + b\cosh(2kd) + f)y^3}{6L} - \rho g y^2\left(\frac{L}{2} - \frac{y}{3}\right)$$

Beam 2-3: $0 \le x \le x_1$

$$M(x) = -M_2 + V_2 x - \rho_m g x_3 \frac{x^2}{2} - \rho_m g c H\left(x - \frac{c}{2}\right)$$

Beam 3-4: $0 \le y \le L$

$$M(y) = -M_3 + N_3 y - \rho g \frac{y^3}{3}$$

Beam 1-4: $0 \le x \le x_1$

$$M(x) = M_1 - V_1 x - (\rho_m x_4 - \rho L) g \frac{x^2}{2} + R_{1y} x$$

Applying the global equilibrium conditions for the whole frame: $\Sigma F_x = 0,\ \ \Sigma F_y = 0,\ \ \Sigma M = 0$, the support reactions are expressed in terms of the variable vector *x* by:

$$R_{1y} = (\rho_m x_4 - \rho L) g \frac{x_1}{2} - (a\cosh(kd) + b\cosh(2kd) + f)\frac{L^2}{3x_1} + \rho_m g x_3 \frac{x_1}{2} + \rho_m g x_5 L + \rho_m g H c\left(1 - \frac{c}{2x_1}\right)$$

$$R_{4y} = (\rho_m x_4 - \rho L) g \frac{x_1}{2} + (a\cosh(kd) + b\cosh(2kd) + f)\frac{L^2}{3x_1} + \rho_m g x_3 \frac{x_1}{2} + \rho_m g (x_1 - x_2 - x_5) L + \rho_m g H \frac{c^2}{2x_1}$$

$$R_{1x} - R_{4x} = (a\cosh(kd) + b\cosh(2kd) + f)\frac{L}{2}$$

Applying the local equilibrium conditions for each beam:

<u>Beam 2-3:</u>

$$\begin{cases} V_2 + V_3 = \rho_m g x_1 x_3 + \rho_m g c H \\ N_2 - N_3 = 0 \\ M_2 - M_3 + V_3 x_1 = \frac{1}{2}\rho_m g x_3 x_1{}^2 + \frac{1}{2}\rho_m g c^2 H \end{cases}$$

<u>Beam 1-2:</u>

$$\begin{cases} V_1 - V_2 = \rho_b g x_5 L \\ N_2 - N_1 = \dfrac{\rho g L^2}{2} + (a\cosh(kd) + b\cosh(2kd) + f)\dfrac{L}{2} \\ M_1 - M_2 + N_2 L = (a\cosh(kd) + b\cosh(2kd) + f)\dfrac{L^2}{3} + \dfrac{\rho g L^3}{6} \end{cases}$$

<u>Beam 1-4:</u>

$$\begin{cases} V_1 + V_4 = R_{1y} + R_{4y} - (\rho_b x_4 - \rho L) g x_1 \\ N_4 - N_1 = R_{1x} - R_{4x} \\ M_4 - M_1 - V_4 x_1 = (\rho_b x_4 - \rho L)\dfrac{g x_1^2}{2} - R_{4y} x_1 \end{cases}$$

Castigliano's theorem is applied in beam 1-4 on the node1 and on the node 4, and beam 1-2 on the node 1; which give 3 new equations to complete the system. (The vertical displacement of the nodes 1 and 4 are equal to zero since it simply supported)

$$\begin{cases} \dfrac{M_1 L^2}{2} + \dfrac{N_1 L^3}{3} + \dfrac{7\rho g L^5}{120} + (a\cosh(kd) + b\cosh(2kd) + f)\dfrac{L^4}{30} = 0 \\ -\dfrac{M_1}{2} + \dfrac{V_1 x_1}{3} + (\rho_b x_4 - \rho L) g \dfrac{x_1^2}{8} - R_{1y}\dfrac{x_1}{3} = 0 \\ -\dfrac{M_4}{2} + \dfrac{V_4 x_1}{3} + (\rho_b x_4 - \rho L) g \dfrac{x_1^2}{8} - R_{4y}\dfrac{x_1}{3} = 0 \end{cases}$$

Finally, it ends up with a system of 12 variables with 12 equations, where these 12 variables ($Ni,\ Vi,$ and Mi) are determined in terms of the breakwater geometrical dimensions x_1, x_2, x_3, x_4, x_5.

The next step in this structural part, after determining the internal efforts and moments, is to develop the expressions of the bending stresses, and the deflections, in order to present them as new constraints needed to be respected in design. Having the bending moments calculated before in terms of $Ni,\ Vi,$ and Mi; the vertical displacements and the bending stresses can be easily deduced based on the following:

$EIy'' = M(x)$, where y'' is the second derivative of the beam deflection, *E* is the Young Modulus of the inside material, *I* is the moment of Inertia of the corresponding beam.

$\sigma = \dfrac{Me}{2I}$, where *e* is the beam thickness

The deflections' constraints are expressed as follows:

$$f_3(x_2,x_3,x_4,x_5) = \frac{1}{EI_{23}}[-\frac{\rho_m g x_3}{24}x^4 + \frac{V_2}{6}x^3 - \frac{M_2}{2}x^2 - \frac{\rho_m g c^2 H}{4}x^2 - \frac{M_2 x_1}{2}x - \frac{v_2 x_1^2}{6}x + \frac{\rho_m g x_3 x_1^3}{24}x + \frac{\rho_m g c^2 H x_1}{4}x]$$

$$f_4(x_2,x_3,x_4,x_5) = \frac{1}{EI_{14}}[-\frac{(\rho_m x_4 - \rho L)g}{24}x^4 - \frac{V_1}{6}x^3 + \frac{R_{1y}}{6}x^3 + \frac{M_1}{2}x^2 - \frac{M_1 x_1}{2}x + \frac{V_1 x_1^2}{6}x + \frac{(\rho_m x_4 - \rho L) g x_1^3}{24}x - \frac{R_{1y} x_1^2}{6}x]$$

$$f_5(x_2,x_3,x_4,x_5) = \frac{1}{EI_{12}}\left[\frac{\rho g}{120}(1 - \frac{H}{L})y^5 - \frac{\rho g L}{24}y^4 - \frac{N_1}{6}y^3 - \frac{M_1}{2}y^2\right]$$

The bending stresses' constraints are expressed as follows:

$$f_6(x_2,x_3,x_4,x_5) = \left[M_1 - V_1 x - (\rho_m x_4 - \rho L) g \frac{x^2}{2} + R_{1y} x\right]\frac{x_4}{2I_{14}}$$

$$f_7(x_2,x_3,x_4,x_5) = \left[-M_2 + V_2 x - \rho_m g x_3 \frac{x^2}{2} - \rho_m g c H\left(x - \frac{c}{2}\right)\right]\frac{x_3}{2I_{23}}$$

$$f_8(x_2,x_3,x_4,x_5) = \left[\begin{array}{l} -M_1 - N_1 y - \dfrac{(a\cosh(kd) + b\cosh(2kd) + f)y^3}{6L} \\ -\rho g y^2\left(\dfrac{L}{2} - \dfrac{x}{3}\right) \end{array}\right]\frac{x_5}{2I_{12}}$$

All the constraints are expressed in long and complicated equations in terms of the four geometrical parameters x_2, x_3, x_4, x_5, characterising the floating breakwater.

Finally, the optimization problem is summarized as follows:

The objective function f_{ob}, establishing the minimum weight of the floating breakwater, has been minimized to design

relative breakwater dimensions according to the following non linear constraints:
Objective function:

$$Min \quad f_{ob}(x_2,x_3,x_4,x_5) = Lx_1 - x_2(L - x_3 - x_4) + Hc$$

Constraints:

$$\begin{cases} f_1(x_2,x_3,x_4,x_5,F) = 0 \\ f_2(x_2,x_3,x_4,x_5) = 0 \\ Max(f_3(x_2,x_3,x_4,x_5)) < 0.01m \\ Max(f_4(x_2,x_3,x_4,x_5)) < 0.01m \\ Max(f_5(x_2,x_3,x_4,x_5)) < 0.01m \\ Max(f_6(x_2,x_3,x_4,x_5)) < 3MPa \\ Max(f_7(x_2,x_3,x_4,x_5)) < 3MPa \\ Max(f_8(x_2,x_3,x_4,x_5)) < 3MPa \end{cases}$$

Aside from the constraints of stability, structural, and floating, it was also necessary to establish some additional geometrical constraints:

$$\begin{cases} x_2 - x_1 < 0 \\ x_3 - x_4 < 0 \\ -x_1 < 0, \quad -x_2 < 0, \quad -x_3 < 0, \quad -x_4 < 0, \quad -x_5 < 0, \end{cases}$$

Using the Matlab optimization toolbox and mainly the function ***fmincon***; which is based on the SQP method (sequential quadratic programming), the problem can be solved to determine the variables x_2, x_3, x_4, x_5, F. (same sea wave and material parameters applied for the fixed bottom breakwater)

$$\begin{cases} x_1 = 8.35m, & x_2 = 6.2m, \\ x_3 = 0.8m, & x_4 = 0.8m \\ x_5 = 0.79m, & F = 2.3\times10^5\, N/1m \end{cases}$$

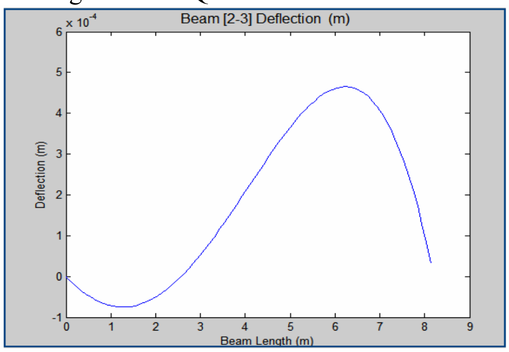
The minimal object function is a non-linear system, any specified feasible solution already subjected to the minimization process can only insure values for the local minimum, and whether these values represent the global minimum must still be confirmed. In fact, the importance of the applied method is realized in its optimality verification, where the conditions of Khun-Tucker are verified:

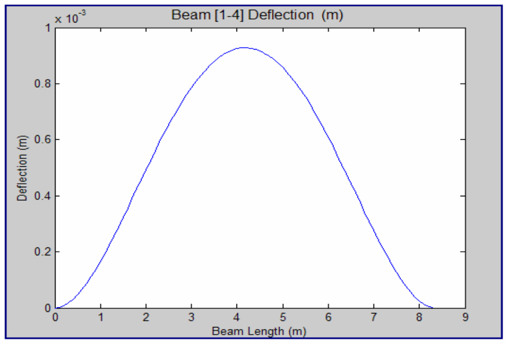
$$\nabla f(x^*) + \sum_{i=1}^{m} \lambda_i \times \nabla f_i(x^*) = 0$$

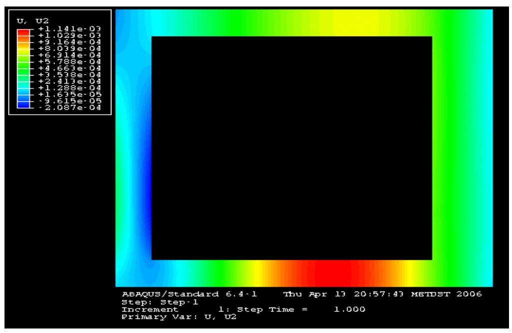
$$\lambda_i \times f_i(x^*) = 0$$

This confirms that our solution is not only a feasible minimum solution, but also an optimal solution verifying the Khun-Tucker conditions of optimality.
In order to validate this analytical calculation, a comparison is realized with a numerical approach using the ABAQUS software, one of the leading softwares in the domain of finite element calculation.
The comparison comprises the deflections and the bending stresses of the most affected and most feeble beams (1-4 and 2-3), where in fact the lower beam is holding the weight and the vertical displacement of all the structure, and the upper beam (2-3) is mainly exposed to the horizontal effort (displacement) and its induced moments caused by the sea waves. Using the Matlab, all the preceding equations (moments, deflections, stresses) can be programmed to yield to explanatory curves defining the real state of the floating breakwater when exposed to sea waves.

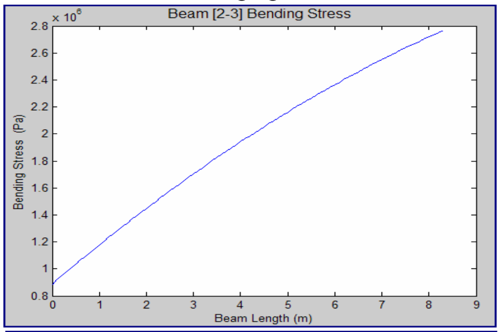
The upper and lower beam deflections are described by the below curves, showing good agreement with the ABAQUS results. First, the upper beam (2-3) has a decreasing-increasing deflection, where it is obviously explained by the subjected moment of the hydrodynamic pressure on the left side of the breakwater causing the decreasing part (*-8x10^{-5}m*) and moreover due to the weight of the vertical wall fixed on the left side of the floating breakwater; where the compression stresses derived form the hydrodynamic pressure on both sides causes a positive deflection attaining a maximum of *4.8x10^{-4}m*. The same trace has been drawn by ABAQUS with a close maximum deflection of *5.7x10^{-4}m.*
The lower beam is supporting all the weight and also the hydrostatic pressure applied at its bottom which is strong enough to cause an upper deflection (*9.28x10^{-4}m*) towards the hollow section and very close to the results given ABAQUS (*1.1x10^{-3}m*), except for the position of the upward maximum deflection where it is approximately located in the middle of the beam in our analytical calculation and shifted smoothly towards the right in ABAQUS.

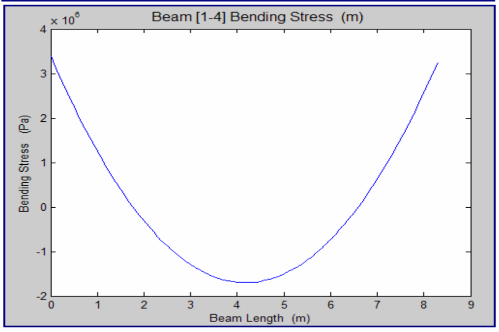


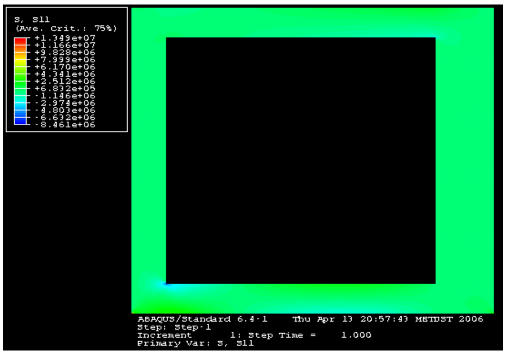




Concerning the bending stresses, a similar comparison to the deflection is realized. The bending stress in the upper beam (2-3) is increasing from *0.9 MPa* on the left side to a value of *2.7MPa* on the right side where it is reaching *2.51 MPa* in ABAQUS. For the lower beam it is decreasing from *3.4 MPa* to a minimum of *-1.7 MPa* and then increasing again. In fact the only value not being respected in the constraints is the bending stress at the node1 and 4, and this is due to the calculation manner of $\max(f_6(x_2, x_3, x_4, x_5))$, where the maximum is located due to the derivative of the function. Despite this, it is normal to have always the large bending stresses located at the fixed supports. In ABAQUS, we can notice that the bending stress is decreasing from *4.34 MPa* to *-1.14 MPa* and then increasing again.







CONCLUSION

As mentioned before, the conventional floating breakwater has low sheltering efficiency and hence it is employed for shore facilities that require a lower level of stability. In this paper, a comprehensive study was performed towards realizing a floating breakwater that can attenuate and withstand strong waves similarly to a fixed bottom breakwater and it ended up by satisfying results. In the table below, a dimensional comparison has been set up between the two breakwaters to show the huge consumed material and efforts in a fixed breakwater that can be saved when moving towards the floating technology (difference in volume is about 22 times).

Type	Dimensions	Volume
Fixed breakwater	x=15m, y=45m	675 m^3/length
Floating breakwater	x_1=8.35 m, x_2=6.2m x_3=0.8m, x_4=0.8m,x_5=0.8	30 m^3/length

In fact, it is not only a problem of volume consuming, but also a structural advantage where the floating breakwater is working approximately in the same stress domain; while in the case of a fixed bottom breakwater (see figure 5) the stress domain is largely varying between the points inside the breakwater, and moreover it is resulting up with huge bending stresses probably causing the collapse of the breakwater in case of very strong waves. This is an additional advantage for the floating breakwater, since the more the inside points are working on closer stresses values the more the extended life of the structure is expected and vice versa.

Finally, we can summarize the following: an analytical model based on the potential flow theory for the wave propagation to compute the pressure of the sea waves on vertical breakwaters has been setup; an extension to structural analysis, based on the mechanics of continuous medium for the stress distribution inside the fixed breakwater and on the frame theory for the floating breakwater, has been studied. In the two cases, a comparison has been realized with a numerical approach to prove the capability of this analytical study. After proving the capability of these analytical models, it can be concluded that a well designed floating breakwater can withstand the same environmental conditions of the fixed one and probably can replace it due to the many advantages listed before.

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