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STUDY OF THE HOMOGENIZATION TECHNIQUE IN A TWO-LAYER CYLINDRICAL MATERIAL

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ABSTRACT

The objective of this study is to analyze the thermo-mechanical behavior of a two-layer cylindrical material, subjected to variable thermal solicitations. The thermo-mechanical coupling is realized by the consideration of the spatiotemporal variations of the temperature in the mechanical problem on one hand, and by means of the variations of the thermal and mechanical properties on the other hand.
The study is led in two dimensions (r, z) on a cylindrical multi-material that consists of two layers subjected to a periodic or constant heat flux on one face, and to an exchange condition on the opposite face. Other faces are supposed to be insulated. We compare two approaches of the problem, the first one is a local approach based on a theoretical study of the thermal and mechanical fields (thermal transfer and constraints) and the second is an approach based on the notion of equivalence of the thermo-physical properties of the tow layers to only one. Thus, this second approach is a homogenization of the multi-material in a single fictitious material possessing the same thermo-mechanical behavior.

1. INTRODUCTION

The need of improvement of the thermal exchanges and the resistance of the thermo-mechanical load lead to the use of new materials having a multi-layers form.
At present, the study of the thermo-mechanical behavior of the multi-materials and more particularly, the analysis of their damage under variable thermal solicitations represent an essential decisive element for numerous designers of materials produced for diverse industrial applications such as engines, reactors and certain boilers etc.
In the scientific field, the existing laws of damage, more particularly those of Lemaitre and Chaboche (1) cannot be applied on multi-materials; they can only be applied on simple and unique materials.
Furthermore, the studies of both layers supposed to be in a perfect contact (2) lead to perfectly theoretical results; for example, they lead to an ideal heat transfer and the constraints are distributed regularly and are discontinuous at the level of the interface between layers.
It is also evident that in a tow- layer material, there is a mutual influence of one material on the other; we cannot have an abrupt variation of the thermal and mechanical properties at the level of the joining. At this level, we have a certain region in which the values of thermal and mechanical properties are supposed to vary between that of the constituted materials.

This reasoning imposes the necessity of the research for a method of correction based on the principle of homogenization of multi-materials in a single equivalent material. This latter has to undergo the study of the damage under thermo-mechanical constraints.
The homogenization approach proposed in (3) replaces the multi-material by a unique material having constant and equivalent thermo-physical properties obtained by means of models that allow us to have similar temperatures in both materials and in equivalent materials also.
Such an approach allows us to translate only the global behavior of the tow-layer material but cannot take account for discontinuities at the level of the interface between the two

layers. The models of damage, based on such homogeneous materials, cannot translate the real behavior of the tow layer-material where the problems of damage are more localized in the interfaces.

In this work, we propose an approach to search for an equivalent material having variable thermo-physical properties according to the space variables. These properties are obtained by means of a polynomial interpolation with convenient orders allowing us to alleviate the effect of the problems of discontinuities.

NOMENCLATURE

Symbols:

C calorific capacity, J/Kg k
E module of elasticity, Pa
e_i thickness of the layer i (i=1,2)
e thickness of the body ($e = e_1 + e_2$)
h coefficient of convection, W/m^2 K
n order of polynomial interpolation
Q density of flux, W/m^2
r radial coordinate, m
T temperature, K
T_0 initial temperature, K
z axial coordinate, m

Greek letters:

ρ density, Kg/m^3
α expansion coefficient, m/K
λ thermal conductivity, W/m K
ν Poisson coefficient
σ constraint tensor, Pa
ε deformation tensor,

2. MODEL DESCRIPTION

The model studied is a tow-layer material consisting of two coaxial cylinders; the internal layer is made by a material different from that of the external layer (steel, copper, aluminum, brass …). The physical properties of these two materials are supposed to be known. The thermal load applied to this system is as follows:
In the internal part, we apply a periodical or constant heat flux.
In the external part, there is an exchange by convection with the environment at the temperature (Ta) and with a convection coefficient h.
Under these conditions the equations of the heat balance are written:

$$(\rho\cdot C)_i\frac{\partial T_i}{\partial t}=\lambda_i\left(\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T_i}{\partial r}\right)+\frac{\partial^2 T_i}{\partial z^2}\right)\qquad i=1,2 \qquad (1)$$

Where i represents the order of the layer.

Initial conditions:

$$T_i(r,z,t)=T_0 \quad à \quad t=0 \quad \forall r,z$$

Internal surface:

$$-\lambda_1\frac{\partial T_1}{\partial r}=Q(z,t)\,,\; r=R_1$$

Interface conditions:

$$T_1=T_2\,,\;\lambda_1\frac{\partial T_1}{\partial \mathrm{r}}=\lambda_2\frac{\partial T_2}{\partial \mathrm{r}}\,,\quad r=R_2$$

External surface:

$$-\lambda_2\frac{\partial T_2}{\partial \mathrm{r}}=h(T(r,z,t)-Ta)\,,\; r=R_3$$

Lateral surfaces:

$$\lambda_i\frac{\partial T_i}{\partial z}=0\,,\; z=0 \text{ and } z=L\,,\; i=1,2$$

On the mechanical plan, the two-layer material is supposed to be fixed at the lateral surfaces $z=0$ and $z=L$. The other surfaces, ($r = R_1$) and ($r = R_3$), are free to be deformed. By neglecting the dynamic effects and the forces of volume, the thermo-mechanical behavior of the material is described by the relations of Duhamel-Neuman:

$$\sigma_{ij}=D_{ijkl}\left(\varepsilon_{ij}-\alpha_{ij}(T-T_0)\right) \qquad (2)$$

For obvious reasons of symmetry, only half of the cylinder is considered.

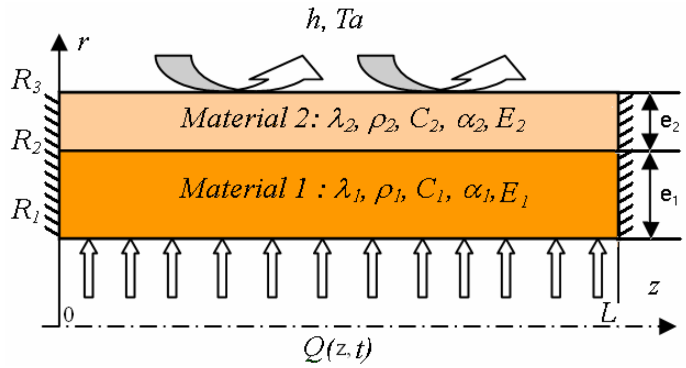

FIGURE 1: THE TOW-LAYER MATERIAL

3. EQUIVALENT MODEL

The research of the equivalent material consists of using a thermal and a mechanical homogeneous model with physical properties that are variable in space and that meet well defined thermal criteria (figure 2). The equivalent suggested material will be described by the following model:

$$(\rho C)(r)\frac{\partial T}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\lambda(r)\frac{\partial T}{\partial r}\right)+\lambda(r)\frac{\partial^2 T}{\partial z^2} \qquad (3)$$

Initial conditions:

$$T(r,z,t)=T_0 \quad \text{à} \quad t=0 \quad \forall r,z$$

Internal surface:

$$-\lambda\frac{\partial T}{\partial r}=Q(z,t)\,,\quad r=R_1$$

External surface:

$$-\lambda\frac{\partial T}{\partial r}=h(T(r,z,t)-Ta)\,,\quad r=R_3$$

Lateral surfaces:

$$\lambda\frac{\partial T}{\partial z}=0\,,\qquad z=0 \text{ and } z=L$$

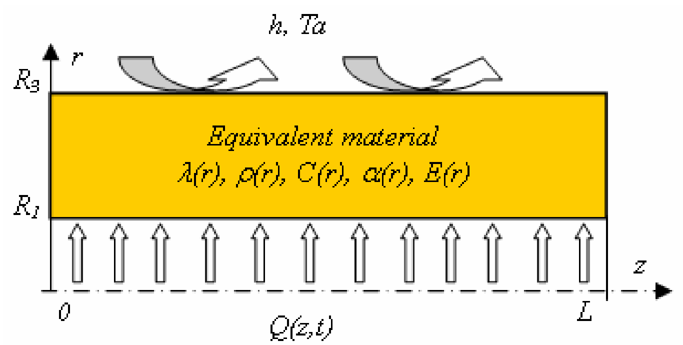

FIGURE 2: EQUIVALENT MATERIAL

The equation of the thermo-mechanical coupling, in matrix form, can be written:

$$\begin{bmatrix}\sigma_{rr}\\ \sigma_{zz}\\ \sigma_{rz}\end{bmatrix}=\frac{E(r,T)}{(1+\upsilon)(1-2\upsilon)}\begin{bmatrix}1-\upsilon & \upsilon & 0\\ \upsilon & 1-\upsilon & 0\\ 0 & 0 & \frac{1-2\upsilon}{2}\end{bmatrix}\left(\begin{bmatrix}\varepsilon_{rr}\\ \varepsilon_{zz}\\ \gamma_{rz}\end{bmatrix}-\begin{bmatrix}\alpha_{rr}(r)\\ \alpha_{zz}(r)\\ 0\end{bmatrix}(T(r,z,t)-T_0)\right) \tag{4}$$

With the mechanical boundary conditions described below.
In our case $\sigma_{rr}=0$ and $\sigma_{zz}\neq 0$

4. NUMERICAL RESULTS

The two systems described above are solved by implicit finite differences. The field of temperature is obtained at each step of time by solving a bloc three-diagonal system with TDMA algorithm. In fact, the thermal effect in axial direction is neglected.
We do a comparison between the heating effect on the double-layered model studied traditionally (by considering a perfectly theoretical joining), and the heating effect on the homogeneous equivalent model.

This comparison is made while varying different geometric and solicitation parameters. The objective is to know when the homogenization is valid and when it is not.
So, we are checking the homogenization technique with the variation of the following parameters:

1- The thickness of the studied body (from 6 mm, to 600 mm).
2- The intensity of the applied heat flux (from 1000 W/m^2, to 10000 W/m^2)
3- The nature of applied heat flux (periodic or constant)
4- The thermal properties of the materials (by modifying the materials studied: steel, copper, brass, aluminum).

The field of the temperatures and the constraints is then obtained by a direct calculation of the thermal and thermo-mechanical equations described above.

In our study, the fixed geometrical properties of the studied model are:

$R_1 = 0.1m$ $\qquad$ $L = 0.6m$

The thermal boundary conditions are:

Ta = 300 K $\qquad$ $h = 6\ W/m^2.K$.

The numerical simulations were carried out with several materials whose properties are summarized in table 1 for the thermal properties and in table 2 for the mechanical properties at three temperatures.

T=20 C	Brass	Copper	Steel	Aluminum
ρ kg/m^3	8600	8930	7854	2770
C J/kg.K	450	385	434	875
λ W/m°K	128	330	60.5	177
α K-1			1,18.10-5	2,3.10-5

TABLE 1: PHYSICAL PROPERTIES OF THE MATERIALS (1)

For the equivalent model, the thermal and mechanical physical properties were obtained by a polynomial approximation according to r in the variation domain limited by the properties of each layer. The degree of this approximation depends on the geometrical, thermal and mechanical properties. It has a direct influence on the difference between the two models.

$$\phi(r)=\sum_{k=0}^{n}a_k r^k\,, \tag{5}$$

Where $\phi=(\rho,c,\lambda,\alpha)$
And n is the order of the approximation.

T (C)	E (MPa) Steel	ν : steel
20 200 600	216000 205000 170000	0.29 0.30 0.315
T (C)	E (MPa) aluminum	ν: aluminum
20 200 500	72000 66000 50000	0.32 0.325 0.35

TABLE 2: VARIATION OF THE CHARACTERISTICS WITH T (1)

For example, in the case of steel (e_1 = 4 cm) and aluminum (e_2 = 2 cm) multi-material, the polynomial approximation of order 8 gives the following interpolation curves for ρ, c, λ and α represented in figures 3, 4 and 5.

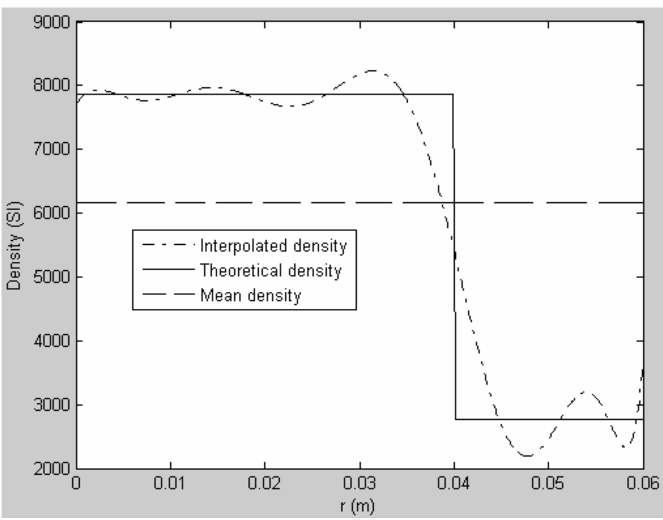

FIGURE 3: CURVES OF INTERPOLATION OF ρ

The variation of the Young modulus according to the temperature for steel and aluminum is given by the relations:

$$Eac\ (T) = -0{,}2469086\ T^2 + 157{,}6837\ T + 175108{,}029 \quad (6)$$

$$Eal\ (T) = -0{,}0990152\ T^2 + 27{,}582\ T + 69395{,}0859 \quad (7)$$

Then, we make the arithmetic interpolation of the Young modulus in the entire model at each temperature. This interpolation is shown in figure 6.

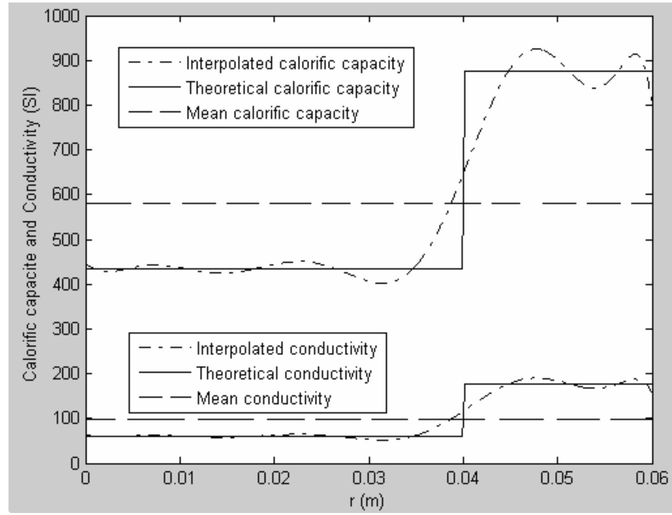

FIGURE 4: CURVES OF INTERPOLATION OF C AND Λ

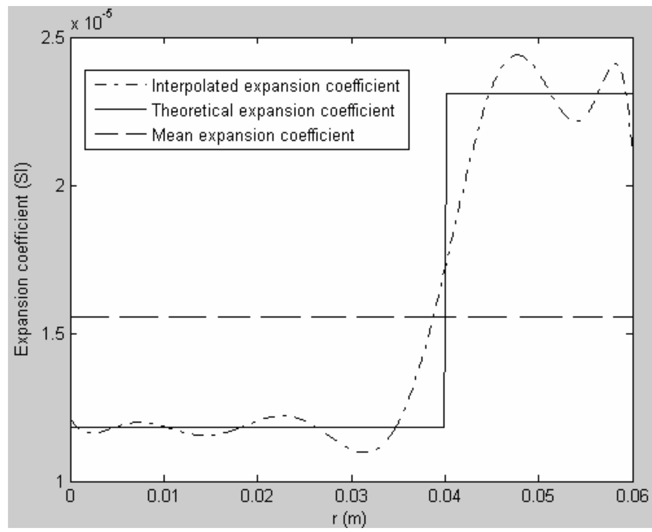

FIGURE 5: CURVES OF INTERPOLATION OF α

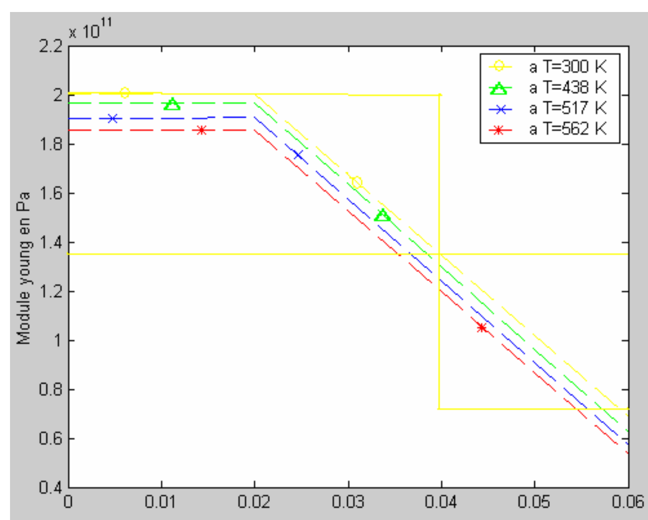

FIGURE 6: CURVES OF INTERPOLATION OF THE YOUNG MODULUS E

It should be noted that in all figures below, the curves are given for a study of the temperature variation in transient theoretical state (continuous curves) and homogeneous (interrupted curves). On each figure, there are 3 curves sometimes clear, sometimes not (according to the zoom), the first one (marked with the circle) corresponds to the point located on the face of application of the flux ($r=R_1$), the second (marked with the triangle) corresponds to the interface between two materials ($r=R_2$), and the third curve (marked with the star) is appropriate for the point subjected to the convection heat transfer ($r = R_3$).

4.1 Influence of Geometry

In this section, the body is subjected to a constant heat flux of 5000w/m^2 on the internal face of the cylinder. The thickness ratio between the two materials (steel and aluminum) is: $e_1/e_2 = 2$.

The other parameters of the system are already defined above.

Here, we are varying the total thickness of the body, according to the following values:

e = 6 mm, 12 mm, 60 mm, 120 mm, 600 mm

Some results obtained here are plotted in figures 7 to 9. These results are obtained with a polynomial interpolation of order 8 and for different mesh values. Figure 10 and 11 represent respectively the relative temperature and constraint error of the homogenization method for different orders of the polynome and for a fixed mesh equal to 12. It can be clearly seen that the relative temperature and constraint errors, for a fixed mesh and for different polynomial orders, increase with the thickness of the two materials. It is found that the more is the thickness; the less exact is the approach of the homogenization, if the mesh remains the same.

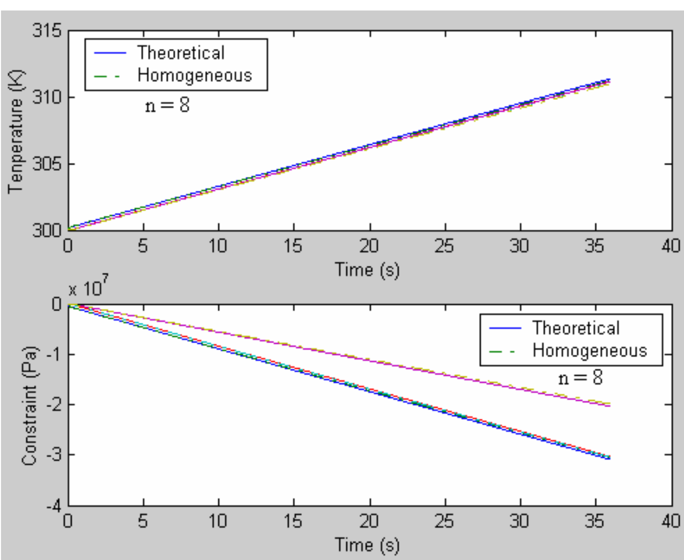

FIGURE 7: COMPARISON OF THE TEMPERATURES AND CONSTRAINT UNDER A CONSTANT LOAD e = 6 MM, MESH = 12

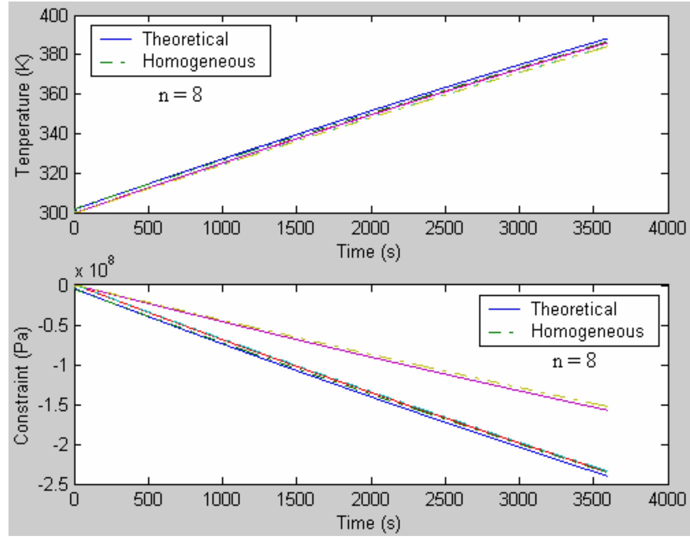

FIGURE 8: COMPARISON OF THE TEMPERATURES AND CONSTRAINT UNDER A CONSTANT LOAD e = 60 MM, MESH = 12

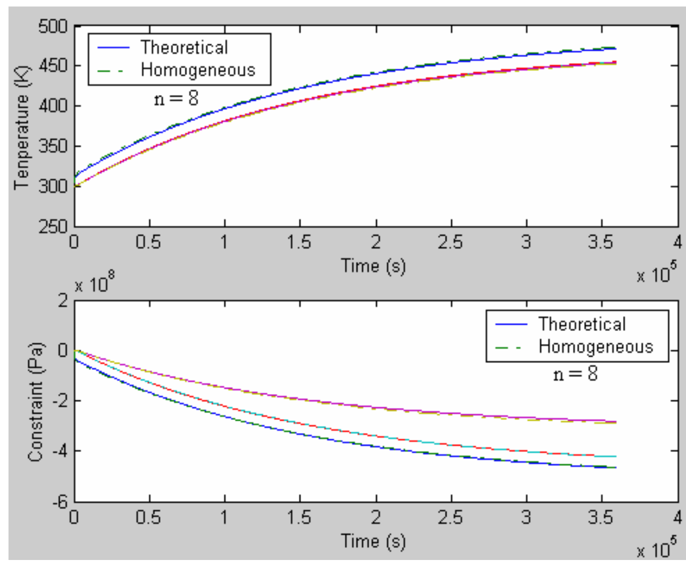

FIGURE 9: COMPARISON OF THE TEMPERATURES AND CONSTRAINT UNDER A CONSTANT LOAD, e = 600MM, MESH = 120

Additionally, we can see, in figures 10 and 11, that the order 6 gives errors larger than that of order 4. It seems that this result is not logical, because we are waiting for small errors when increasing the order of the polynomial interpolation. This problem can be resolved and explained in figures 12 to 15.

To understand what it is happening we run the simulation while fixing the thickness and varying the order of the polynomial interpolation and the meshes. Figures 12 and 13 shows the temperature and constraint errors for a thickness equal to 60 mm, and figures 14 and 15 shows these errors for a thickness equal to 600 mm. We observed that low meshes introduce disordered errors, sometimes low sometimes high. While increasing the mesh, the errors decrease gradually with an increasing order of the polynome.

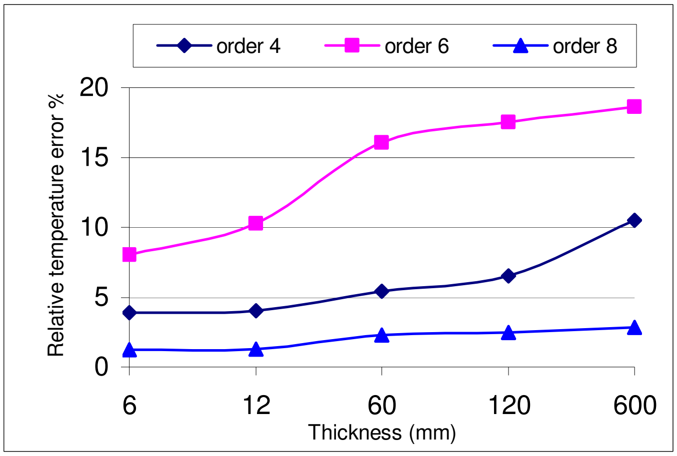

FIGURE 10: TEMPERATURE RELATIVE ERROR VARIATION WITH THICKNESS FOR DIFFERENT ORDERS OF POLYNOMIAL INTERPOLATION, MESH = 12

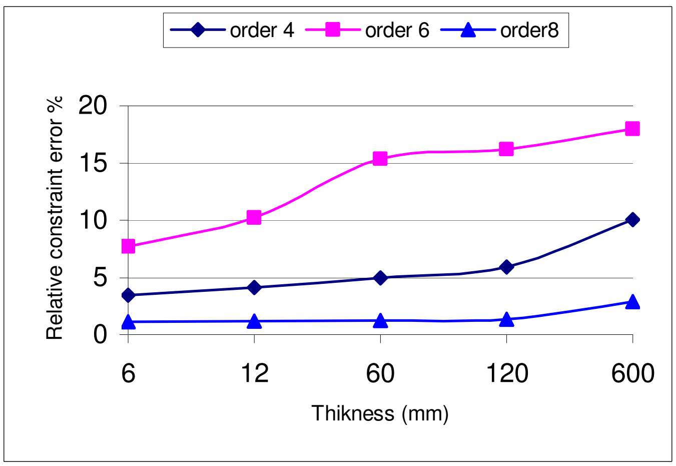

FIGURE 11: CONSTRAINT RELATIVE ERROR VARIATION WITH THICKNESS FOR DIFFERENT ORDERS OF POLYNOMIAL INTERPOLATION, MESH = 12

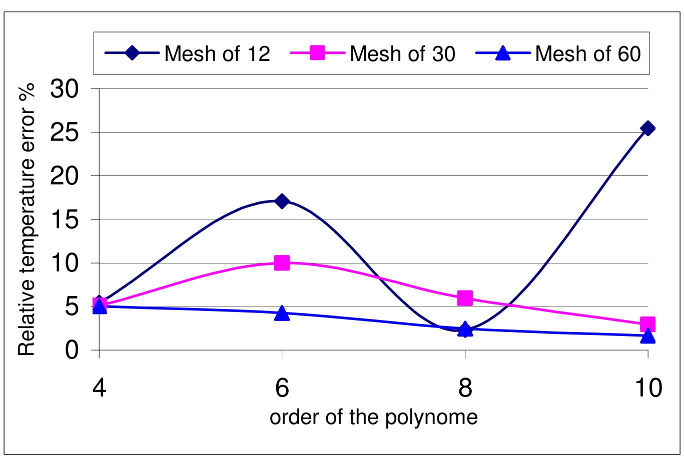

FIGURE 12: TEMPERATURE RELATVE ERROR VARIATION WITH RESPECT TO THE ORDER OF POLYNOMIAL INTERPOLATION FOR DIFFERENT NUMBER OF MESHES, e = 60 MM

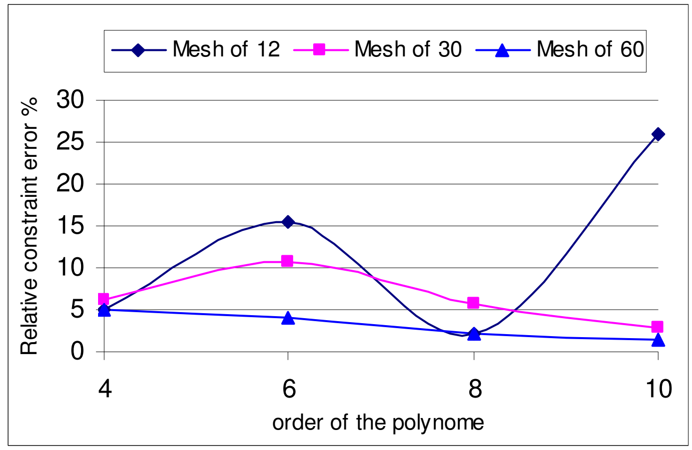

FIGURE 13: CONSTRAINT RELATIVE ERROR VARIATION WITH RESPECT TO THE ORDER OF POLYNOMIAL INTERPOLATION FOR DIFFERENT NUMBER OF MESHES, e = 60 MM

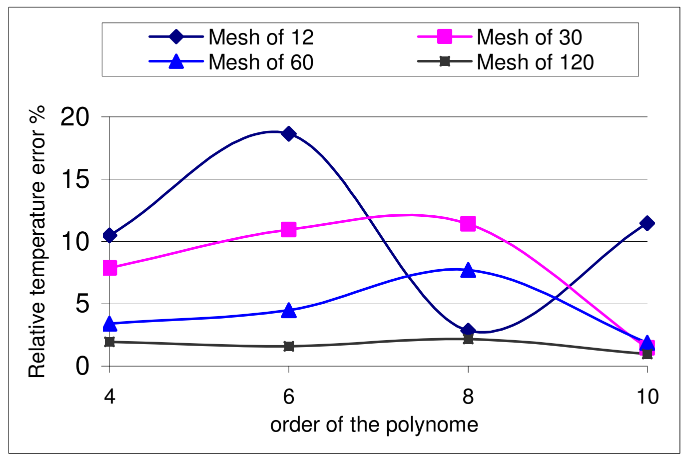

FIGURE 14: TEMPERATURE RELATIVE ERROR VARIATION WITH RESPECT TO THE ORDER OF POLYNOMIAL INTERPOLATION FOR DIFFERENT NUMBER OF MESHES, e = 600MM

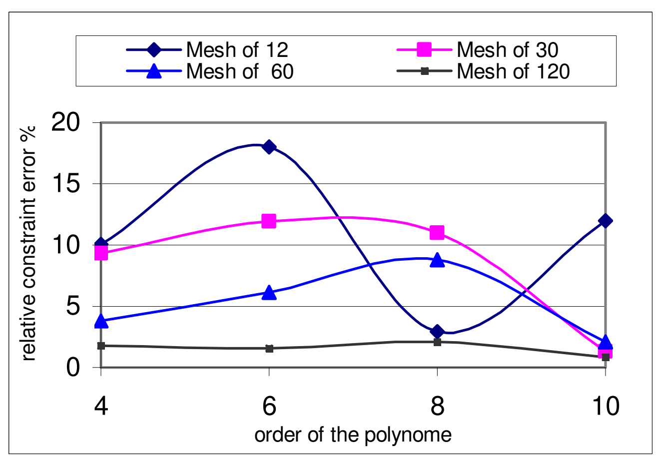

FIGURE 15: CONSTRAINT RELATVE ERROR VARIATION WITH RESPECT TO THE ORDER OF POLYNOMIAL INTERPOLATION FOR DIFFERENT NUMBER OF MESHES, e = 600 MM

Also, we can conclude that high thicknesses (figures 14 and 15) necessitate high meshes in order to obtain equivalency (low errors). For a thickness of 60 mm, a mesh of 60 is sufficient to obtain low errors and normally decreased errors when we increase the order of the polynomial interpolation. But for 600 mm, a mesh of 60 is not sufficient, and we need a mesh of 120 to obtain expected result.

As a conclusion, according to the thicknesses of the two-layer materials, an adequate choice of the mesh and the order of the polynomial interpolation are very important to reach equivalency. For high thicknesses, high mesh and high order of polynomial interpolation are necessary.

4.2 Influence of the flux density

4.2.1. Constant heat flux: In this section, we are applying, first of all, a constant heat flux Q of 1000 w/m^2, 5000 w/m^2 and 10000 w/m^2. The second case (Q = 5000 w/m^2) is studied in the section 4.1.

The boundary conditions imposed and the geometrical properties are indicated in the description of the model. The thickness of the body is fixed to e = 12 mm.

Figures 16 and 17 show that the method of homogenization used is not influenced by the variation of the intensity of the load applied.

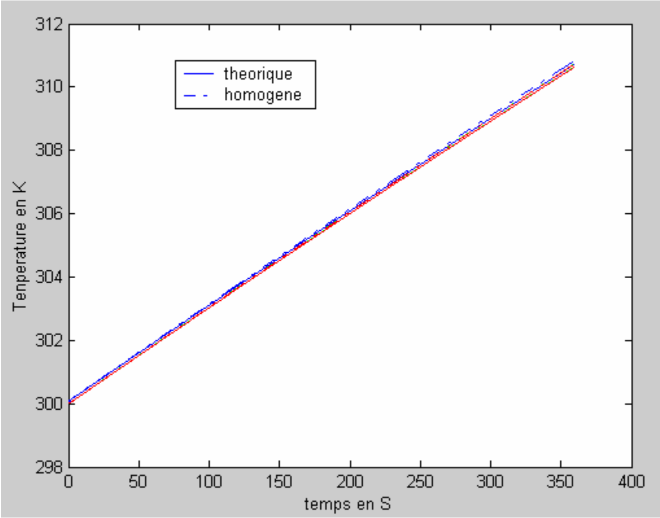

FIGURE 16: COMPARISON OF THE TEMPERATURES UNDER A CONSTANT LOAD: HEAT FLUX = 1000 W/M^2

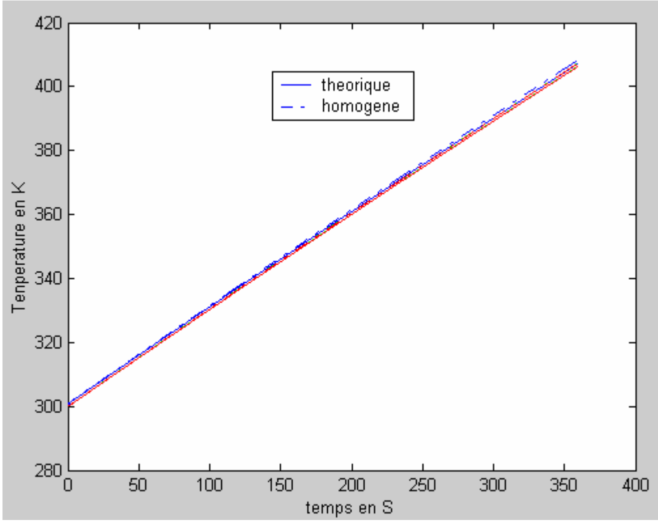

FIGURE 17: COMPARISON OF THE TEMPERATURES UNDER A CONSTANT LOAD, HEAT FLUX = 10000 W/M^2,

4.2.2. Sinusoidal heat flux: Now, we are varying the nature of the applied load, by the application of a sinusoidal periodic flux, defined by the following equation:

$$Q(z,t)=\left(-4Q_0z^2/L^2+4Q_0z/L\right)\left(1+\sin(\omega t)\right),$$
$$Q_0=5000 \quad W/m^2,\ \omega=\pi/20 \quad rd/s \quad (8)$$

Figure 18 shows that the nature of the load applied does not have also any influence on the homogenization method we are using.

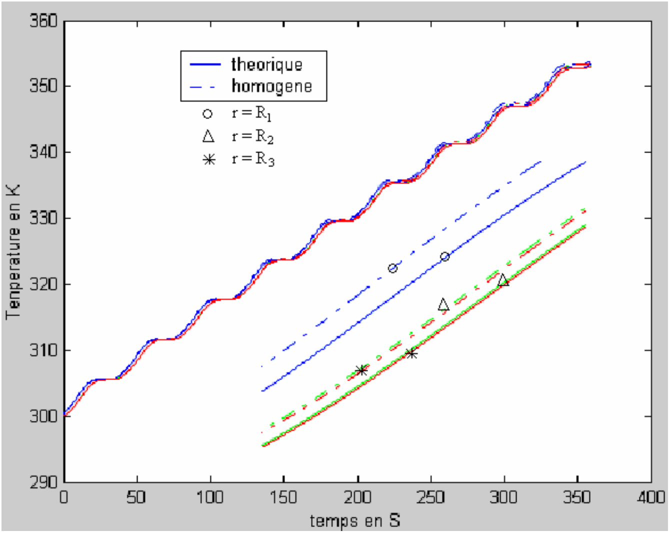

FIGURE 18: COMPARISON OF THE TEMPERATURE, UNDER A VARIABLE LOAD, e = 12 MM

4.3 Influence of material Properties

In this test, we are applying the homogenization method to various couples of material (steel and aluminum, steel and copper, steel and brass) with a constant heat flux.

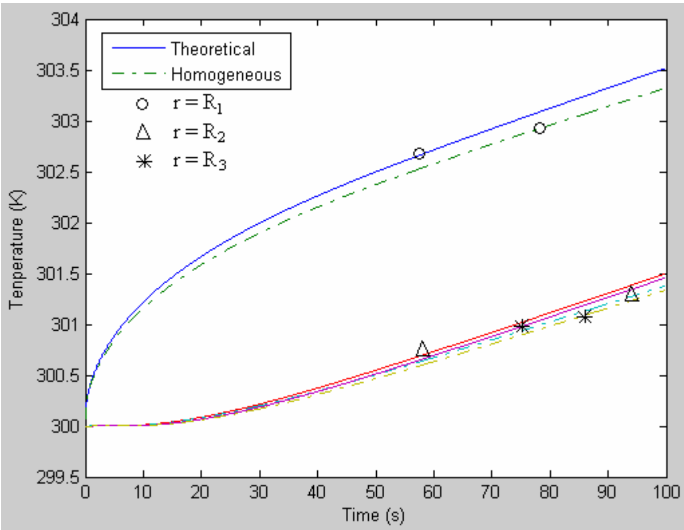

FIGURE19: COMPARISON OF THE VARIATIONS IN THE TEMPERATURES, IN THEORETICAL CASE AND IN HOMOGENEOUS CASE: STEEL AND COPPER (N = 8)

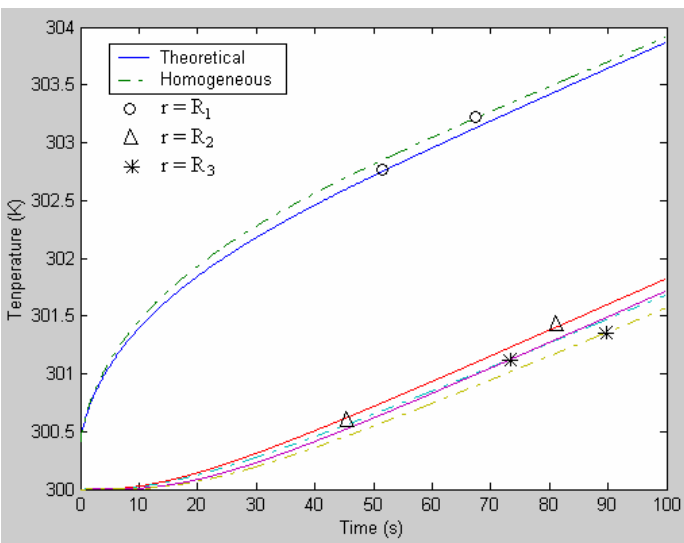

FIGURE 20: COMPARISON OF THE VARIATIONS IN THE TEMPERATURES, IN THEORETICAL CASE AND IN HOMOGENEOUS CASE: STEEL AND BRASS (N = 8)

While applying the homogenization method, we are led to vary the degree of the polynomial interpolation n for each couple of materials in order to obtain equivalence. Using one polynomial interpolation with a fixed degree for all couples of materials is not adequate. The results are very sensitive of the differences between physical properties of the multi-material used.

Figure 21 shows the relative temperature error when using different materials in association with the steel. This figure is plotted to show the relation of the physical properties of these materials (ρ, c and λ) with the error variation. It can be shown that when λ increases, the relative temperature error increases.

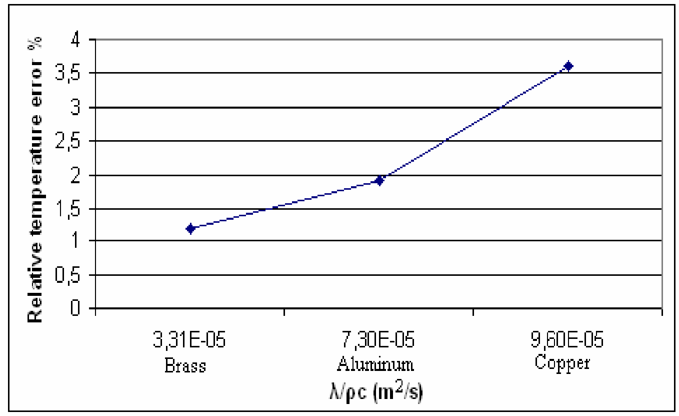

FIGURE 21: RELATIVE TEMPERATURE ERROR FOR DIFFERENT MATERIALS (EXTERNAL LAYER), USED WITH THE STEEL (INTERNAL LAYER) (N = 8)

4.4 Influence of the variation of Young modulus

Now, let us try to compare the constraints obtained in the theoretical and in the homogeneous cases under a constant load. Firstly, with a constant E (figure 22) and then with E function of T (figure 23).

In our simulations, we find that the presence of the Young modulus variation with the temperature increases the approximation error but in a low proportion. For example, in the figures 22 and 23, the relative constraint error between the two models is 2.5825 %, and 4.013 % respectively.

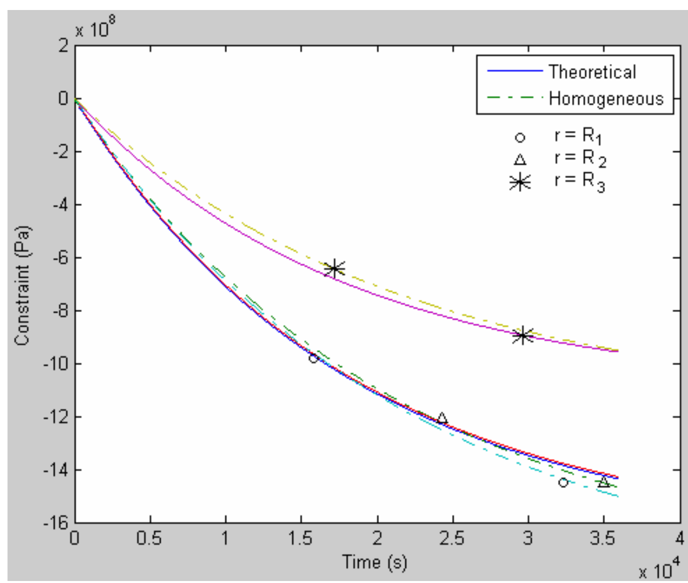

FIGURE 22: COMPARISON OF THE CONSTRAINTS UNDER A CONSTANT LOAD 5000 W/M^2, e = 60 MM, E CONSTANT

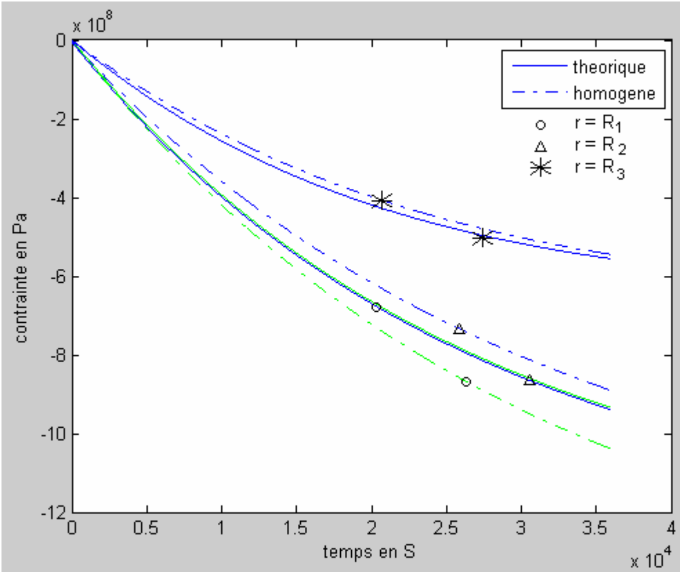

FIGURE 23: COMPARISON OF THE CONSTRAINTS UNDER A CONSTANT LOAD 5000 W/M^2, e = 60 MM, WITH E (T)

This results from the accumulation of interpolation errors of E with the space and with the temperature. So the variation of E with the temperature does not have a great influence on the homogeneous model suggested.

Figure 23 shows also the effect of the variation of E with the temperature on the constraints. It is quite clear that the increase of the temperature in the body increases the difference between the theoretical (constant E) and the corrected constraints (E function of T). Indeed when the temperature increases, the modulus of elasticity decreases in the two materials, and consequently the resistance of the body decreases also in turn.

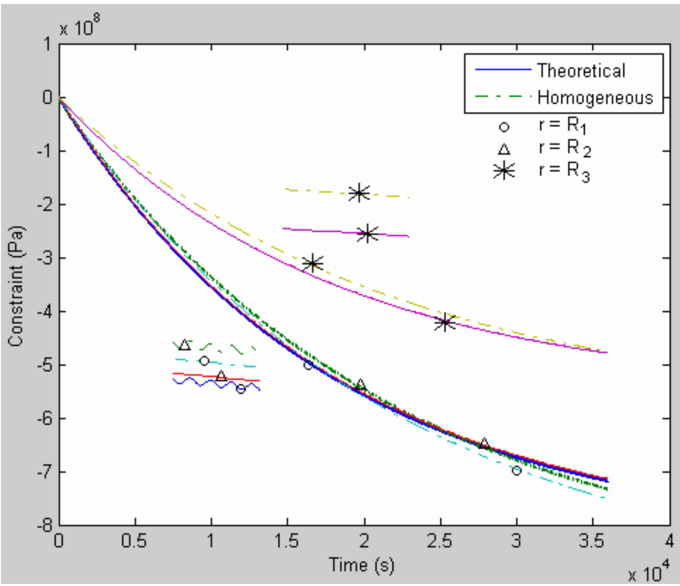

FIGURE 24: COMPARISON OF THE CONSTRAINTS OBTAINED WITH CONSTANT E AND VARIABLE FLOW

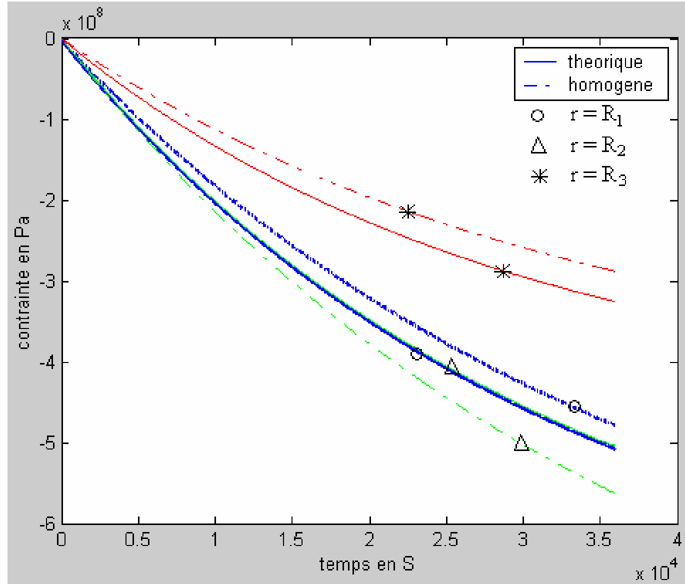

FIGURE 25: COMPARISON OF THE CONSTRAINTS OBTAINED WITH E (T) AND VARIABLE FLOW

For a sinusoidal applied load, we obtain also the same result, (for constant *E* figure 24: average relative constraint error = 2.5826, for E (T), figure 25: average relative constraint error = 4.1275). Consequently, we can conclude that the proposed homogenization model remains valid whatever is the nature of the load and even with the variation of the mechanical properties with temperature E (T).

6. CONCLUSION

The concept of multi-material homogenization is useful in the analysis of the systems and their damage which, for the majority of the models of damage, is based on the concept of equivalent medium.

In this work, we studied a homogenization technique and we showed its validity by being based on fixed thermal and mechanical criteria. We found that the thickness of the body studied and the nature of materials studied have an influence on the interpolation technique used, and it is necessary to adjust the degree of polynomial interpolation well according to the thicknesses and to the nature of the multi-materials.

In the same way, we showed the variation of the elasticity modulus according to the temperature and its effect on the weakening of materials.

The essential goal of the study being always to calculate the local damage in the multi-material, this work is under analysis.

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