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MODELLING WAVE-INDUCED PRESSURE ON BREAKWATERS

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ABSTRACT
Nowadays, modelling and computer simulation are used in structure design to reduce the use of experimental investigations and laboratory tests required in a real layout structure. Using the potential-flow theory, the hydrodynamic pressure deduced from sea wave's propagation has been computed based on the non linear theory of Stocks. The extension to the vertical breakwater behaviour due to such excitations is indicated in this paper. Analytical expressions for functional performance variables of wave reflection by the vertical breakwater, and stress distribution inside the breakwater have been obtained. These expressions have been numerically verified to demonstrate the capability of this analytical model.

KEY WORDS
Analytical modelling, wave forces, vertical breakwater, wave structure interaction, stress distribution, Matlab.

1-Introduction

Coastal structures are designed mainly to provide protection by reflection and/or dissipation of wave energy. Rubble-mound breakwaters have been extensively used for sheltering harbours. However, especially during the last decade, innovative vertical structures may often represent a better alternative than rubble mound breakwaters. In general, caisson-type breakwaters may improve hydraulic performance, total costs, quality control, environmental aspects, construction time and maintenance [1]. Most of these caisson-type structures are monolithic structures with impermeable vertical walls which constitute the subject of this study.
Although the protection of marine structures has been extensively studied in recent years, understanding of their interaction with waves, marine structures and the seabed is far from complete [3]. Damage of marine structures still occurs from time to time, with two general failure modes evident. The first mode is that of structural failure, caused by wave forces acting on and damaging the structure itself. The second mode, which has attracted many of the scientists (Biot-1941; Jeng 1997; Mitzutani 1998), is that of foundation failure caused by liquefaction or erosion of the seabed in the vicinity of the structure, resulting in collapse of the structure; where the work of this paper is mainly concentrated on the structure failure due to the lack of knowledge in this domain. Moreover, identifying these forces acting on the breakwater yields not only for its loading endurance study, but also for optimizing the exterior design and the inside material type in future work. The physical understanding and computation of wave–structure interaction, one of the most important hydrodynamic processes in both coastal and offshore engineering, are crucial to assess wave impacts on structures as well as structural responses to wave attacks. Traditionally, the estimation of wave loads on a structure is often done by either empirical approach [2] (ex: Morison equation, Sainflou, Hiroi, Goda, Svendson…) or a computational approach. The empirical formulas are simple but crude and will not be able to provide detailed and accurate information about pressure distribution on a structure. The computational approach can be further divided into two types: the Laplace equation solver for potential flows and the Navier–Stokes Equations (NSE) solver for viscous flows, where the latter is used for simulation of wave–structure interaction during which both vortices and turbulence may be present. In this paper it is interesting to treat the Laplace equation, where the potential-flow problem is solved by imposing the boundary conditions and then a structural modelling is developed to define the pressure distribution inside the breakwater.

2-Methodology

This part identifies the methodology followed in the analytical modelling of the waves and their induced pressures exerted on vertical breakwaters and finally the behaviour of the breakwater due to wave-structure interaction. It is interesting to consider the case of a vertical breakwater appearing in ports' constructions far from the shore, at a constant depth, and at a fixed point. Then, the problems of wave's propagation over a varying bathymetry and shallow water consequences are eliminated.

2.1 Wave modeling

A cartesian coordinate system *Oxyz* is employed, where *Oxy* coincide with plane of the free surface at rest, *Oz* directed positive upwards, and *Ox* directed positive in the direction of propagation of the waves. The incident wave propagates in a straight line in the direction defined by the angle γ, formed with the *Ox* axe. In this study, it is supposed that the waves can strike the breakwater in a perpendicular direction to obtain the maximum pressure applied by the waves on the breakwater, in order to study the dangerous case in the construction of a breakwater. Then, the angle is taken as $\gamma = 0$ (incident wave normal to the breakwater) and the movement is reduced to two dimensions as in figure 1.

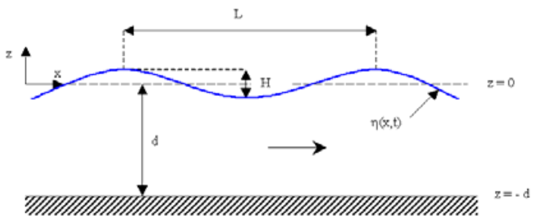


Figure 1 Wave notations

The fluid motion is defined as follows: Let t denote time, x and z the horizontal and vertical coordinates, respectively, and η the free-surface elevation above the still water level. The high values of the density and sound velocity in water render the compressibility effects negligible in sea water, so it is considered incompressible. The fluid is considered also irrotational. Then, the fluid motion can be described by a velocity potential, Φ, related to the velocity $\vec{U}(u,w)$.

$\overrightarrow{rot}(\vec{U})=\vec{0} \Rightarrow \vec{U}=\overrightarrow{grad}(\Phi)$, where $u=\frac{\partial\Phi}{\partial x}$ and $w=\frac{\partial\Phi}{\partial z}$.

Once the parameters characterizing the sea waves are known (Length of wave L, Period T, Height H), a model is needed to study the waves' propagations and transforms their evolution into loads on the breakwater. It is a strict study based on the fundamental physical principles of the conservation of momentum and mass. The combination of the equation of momentum conservation and that of mass, yields to the well known equation, Bernoulli-Lagrange, which constitutes the essential equation to determine the field of wave's pressure.

$$\frac{\partial\Phi}{\partial t}+\frac{1}{2}(grad\Phi)^2+\frac{P(x,z,t)}{\rho}+gz=Q(t)$$

In general, the study of marine structures' behaviours due to waves' propagations is mostly made as part of a linear theory [4], where the interest in this paper is to orient the work towards the non linear approximation (Stokes 2nd order expansion), which yields to a clarified view of the efforts in an enlarged domain of frequencies and moreover the linear wave theory is not expected to have outstanding agreement with the real and experimental data.

It is clear that if Φ is known throughout the fluid, the physical quantities pressure and velocity) can be obtained from Bernoulli's equation. The boundary value problem is then defined as follows:

$\nabla^2\Phi=\Delta\Phi=0$ Laplace equation in the fluid domain;

$\left(\frac{\partial\Phi}{\partial z}\right)_{z=-d}=0$ Condition at the sea floor;

$\left(\frac{\partial\Phi}{\partial n}\right)_{x=0}=0$ Kinematic condition at the solid boundary;

$\left(\frac{\partial\eta}{\partial t}+\frac{\partial\Phi}{\partial x}\frac{\partial\eta}{\partial x}-\frac{\partial\Phi}{\partial z}\right)_{z=\eta}=0$ Kinematic condition at the free surface;

$\left(\frac{\partial\Phi}{\partial t}+\frac{1}{2}\left(\left(\frac{\partial\Phi}{\partial x}\right)^2+\left(\frac{\partial\Phi}{\partial z}\right)^2\right)+g\eta\right)_{z=\eta}=Q(t)$ Dynamic equation at the free surface;

The equation of Laplace expresses the mass conservation; the sea bottom condition expresses the impermiability of the sea bed where the normal component of the velocity is zero; the kinematic condition at the solid boundary (breakwater, $x=0$), expresses the static condition of the breakwater (wave reflection) where $\vec{n}$ is the outward normal direction of the solid boundary; the kinematic condition on surface, $z=\eta$, expresses that a fluid particle at the surface should remain there at all times, while the dynamic condition expresses that the pressure on the free surface is zero. The used method for the nonlinear theory (Stockes 2nd order expansion), called perturbation method [5], consists of developing the different variables into power series depending on a parameter $\varepsilon=\frac{H}{L}$, where the linear theory constitutes the first order yielding exact solutions only for waves with infinitesimal amplitudes.

$$\Phi=\varepsilon\Phi_1+\varepsilon^2\Phi_2+\varepsilon^3\Phi_3+.........+\varepsilon^n\Phi_n\}$$

By considering the amplitudes of the oscillations of the free surface to be small, the terms are then evaluated on the free surface depending on $\eta(x,t)$ due to Taylor series.

$$\Phi(x,\eta)=\Phi(x,0)+\eta\left(\frac{\partial\Phi}{\partial z}\right)_{z=0}+.........+\frac{\eta^n}{n}\left(\frac{\partial^n\Phi}{\partial z_n}\right)_{z=0}$$

The developments are limited to the second order of the camber ε so: $\Phi=\varepsilon\Phi_1+\varepsilon^2\Phi_2$ and $\eta=\varepsilon\eta_1+\varepsilon^2\eta_2$. It is convenient to determine $\Phi_2(x,z,t)$ and $\eta_2(x,t)$ knowing Φ_1 and η_1 (linear case), Then the boundary conditions for the free surface for $z=\eta(x,t)$, are transformed into perturbation series. Solving for the 1st order expansion (linear theory)

$$\Phi_1=\mathrm{Re}\left\{-i\frac{Hg}{2\omega}\frac{ch[k(z+d)]}{ch(kd)}\exp i(kx-\omega t)\right\}\ ,\ \eta_1=\mathrm{Re}\left\{\frac{H}{2}\exp i(kx-\omega t)\right\}$$

(Where $k=2\pi/L$ designates the wave number and ω the frequency). The nonlinear approximation is achieved by substituting for the first order in the perturbation series:

$$\Phi(x,z,t)=\mathrm{Re}\left\{-i\frac{Hg}{2\omega}\frac{ch[k(z+d)]}{ch(kd)}\exp i(kx-\omega t)\right\}-\frac{\pi gH^2}{4L}\frac{t}{sh(2kd)}$$
$$+\mathrm{Re}\left\{-i\frac{3}{8}\left(\frac{H}{2}\right)^2\frac{gk}{\omega}\frac{ch[2k(z+d)]}{sk^3(kd)ch(kd)}\exp 2i(kx-\omega t)\right\}$$

This expression of velocity potential describes the physical properties of the waves in the absence of any structure, where the reflection phenomenon must be taken into consideration during the collision of the waves by the breakwater. Then, a reflected wave identical to the incident one is created but in the opposite sense.

$\Phi_r(x,z,t) = r \times \Phi_i(-x,z,t)$

Where r designates the reflection coefficient (coefficient of amplitude reduction), the superposition of the incident and reflected velocity potentials creates a global wave system [7] whose velocity potential is defined as: $\Phi_T = \Phi_i + \Phi_r$.

Moreover, the extremity of the breakwater involves the diffraction of the waves and hence concentric circles are formed around its extremity. Considering a semi-infinite breakwater, eliminates this phenomenon and keeps the problem in the domain of wave reflection only; where the global potential velocity describing the problem is maintained as expressed above. The substitution of this value for the velocity potential (Φ_T) in the Bernoulli-Lagrange equation implies the expression of the pressure distribution (pressure at any point in the fluid domain.) in the case of wave-breakwater interaction, where all the waves are reflected by the breakwater (no diffraction or transmission).

$$P(x,z,t) = -\rho g z + \mathrm{Re}\left\{\begin{array}{l}\frac{1}{2}\rho g H \frac{ch[k(z+d)]}{ch(kd)} \\ \left[\exp i(kx-\omega t) + r\exp i(-kx-\omega t+\beta)\right]\end{array}\right\}$$
$$+ \mathrm{Re}\left\{\begin{array}{l}\frac{3}{4}\rho g H \frac{\pi H}{L}\frac{1}{sh(2kd)}\left[\frac{ch2k(z+d)}{sh^2 kd} - \frac{1}{3}\right] \\ \left[\exp 2i(kx-\omega t) + \left(r^2+r\right)\exp 2i(-kx-\omega t+\beta)\right]\end{array}\right\}$$
$$+ \mathrm{Re}\left\{r\rho H^2\omega^2 \exp i(-2\omega t+\beta)\right\} - \frac{1}{4}\rho g H \frac{\pi H}{L}\frac{(r+1)}{sh(2kd)}\left[ch2k(z+d)-1\right]$$

2.2 Vertical Breakwater modelling

There is no experience at hand to premeditate the destruction failures or damage of breakwater at design stage itself, and this return to the lack of detailed structural studies. The analytical method proposed in this paper for the analysis of structural construction of conventional breakwaters is based on predicting the hydrodynamic forces (induced from the waves), and then analyzing the stress repartition in the breakwater by considering the hydrodynamic forces and the inertia forces due to rigid-body motions as external forces. This study is assimilated to a mechanical problem in a continuous medium, which consists to determine analytically the stress tensor $\sigma(M)$ on each point M in the solid of the studied structure [6],[8]. The general method used in elasticity theory to determine the stress distribution in the interior of an elastic body, is based on proposing the form of the stress tensor that must satisfy the equilibrium equations, all the boundary conditions, and the compatibility equations. The disadvantage of this method is that it needs a lot of function trials (especially with such non uniform equation of hydrodynamic pressure), since there are hundreds of functions that satisfy the above conditions but not all them describe the real state of the problem. For the simplicity of calculations, it is recommended to divide the problem over two parts: the hydrodynamic forces developed from the waves' propagations and the hydrostatic forces developed from water depth.

Hydrodynamic pressure

The exerted pressure by waves on the vertical breakwater is deduced from the computed fluid problem in the first section. This hydrodynamic pressure has a complicated expression different from the hydrostatic one that is linear, its repartition over the breakwater has a curved shape (obtained using Matlab); where its maximum is around the still water level and it decreases to zero at the top of the breakwater (with the wave height) and also decreases with water depth (figure 2). Fixing $x = 0$ (exterior breakwater surface), and the phase angle $\beta = 0$ (vertical impermeable wall, [9]), the pressure distribution over the vertical breakwater is obtained.

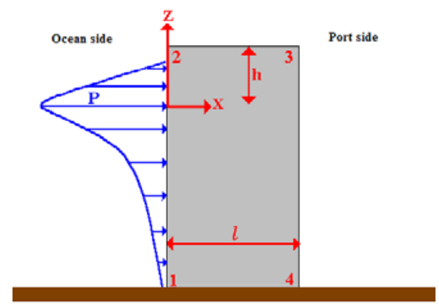


Figure 2 Hydrodynamic pressure distribution over the breakwater

The hydrodynamic pressure exerted by the waves on the breakwater is acting on the exterior surface of the breakwater due to the assumption that all the waves propagating from the ocean side are totally reflected outside the port (no transmission); and hence it can be simply deduced that there are no dynamic pressure acting on the interior surface of the breakwater due to the absence of waves' propagations inside the port. It can be written as follows:

$$P = a\cosh k(z+d) + b\cosh 2k(z+d) + f$$

$$a = \frac{\rho g H}{2}\frac{(r+1)}{chkd}\cos(\omega t)\ ,\quad b = \frac{\rho g \pi H^2}{4Lsh2kd}\left[\frac{(3r^2+3r+3)\cos(2\omega t)}{sh^2 kd} - r - 1\right]$$

$$f = \frac{\rho g \pi H^2}{4Lsh2kd}\left[\left(-r^2-r-1\right)\cos(2\omega t) + r + 1\right] + \rho H^2\omega^2 r\cos(2\omega t)$$

It is reduced to an equation with hyperbolic functions of z (height), where the other variables independent of the altitude are collected together in the terms a, b, and f .

The hydrodynamic stress's form is chosen as:

$$\sigma_{xd} = A_1 z\cosh k(z+d) + B_1\sinh k(x-l) + C_1\cosh 2k(z+d)$$
$$\sigma_{zd} = A_2 z\cosh k(z+d) + B_2(z-h)^3\sinh k(x-l)$$
$$+ C_2 x^3\cosh 2k(z+d) + D_2(x-l)^3\cosh k(z+d)$$
$$\tau_{xzd} = A_3 z\cosh k(z+d) + B_3 z^2\cosh k(x-l)$$
$$+ C_3 x^2\cosh 2k(z+d) + D_3(x-l)^2\cosh k(z+d)$$

where terms $A_1, B_1, C_1, A_2, B_2, C_2, D_2, A_3, B_3, C_3, D_3$ are to be determined when applying the equilibrium, boundary, and compatibility equations.

a-Equilibrium conditions:

$$\vec{div}\,\sigma + \vec{f_v} = \vec{0} \quad , \quad \begin{cases} \dfrac{\partial \sigma_x}{\partial x} + \dfrac{\partial \tau_{xz}}{\partial z} = 0 \\ \dfrac{\partial \tau_{xz}}{\partial x} + \dfrac{\partial \sigma_z}{\partial z} = \rho_b g \end{cases}$$

b-Boundary conditions: The condition for the ocean side (1-2) is : $\overset{=}{\sigma}.\vec{n} = \vec{\sigma_f}$, where $\vec{n}$ is the normal to the side (1-2), which is equal to (-1,0)

The constraint vector $\overset{=}{\sigma}.\vec{n}$ is equal to the surface force $\vec{P}$

$$\begin{bmatrix} \sigma_x & \tau_{xz} \\ \tau_{xz} & \sigma_z \end{bmatrix} \text{x} \begin{bmatrix} -1 \\ 0 \end{bmatrix} = \begin{bmatrix} p \\ 0 \end{bmatrix}$$

For the port side, there is absence of any type of dynamic pressure, then on $x = l$: $\overset{=}{\sigma}.\vec{n} = \vec{0}$,and also for the upper side of the breakwater, no forces are exerted, then on $z = h$: $\overset{=}{\sigma}.\vec{n} = \vec{0}$

c-Compatibility equation: The problem is statically undetermined by the above conditions alone and to solve such problem, elastic deformations must be introduced. The mathematical expression of the compatibility equation of the stress distribution with the existence of the continuous functions U and V , defines the deformation state. This compatibility equation, represented in a differential equation, assures the existence of the deformation functions and satisfies the boundary conditions also. (the 6 compatibility equations are reduced to one equation in the case of 2D)

$$\frac{\partial^2 \varepsilon_x}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial x^2} = \frac{\partial^2 \gamma_{xz}}{\partial x \partial z} , \qquad \varepsilon_x = \frac{\partial U}{\partial x} , \varepsilon_z = \frac{\partial V}{\partial z} , \gamma_{xz} = \frac{\partial U}{\partial z} + \frac{\partial V}{\partial x}$$

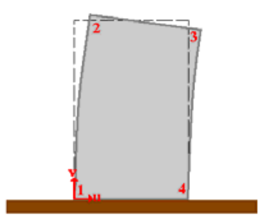


Figure 3 Fixed support stability conditions

The integration of the above expressions yields to determine the deformations' functions for each point in the solid; where q and r (integration constants) are two functions chosen to satisfy the sufficient conditions to identify the problem:

$q(z) = m \sinh k(z + d)$, m is a variable to be determined

$r(x) = 0$, chosen equal to zero for the lack of any additional boundary condition. The boundary conditions applied at the bottom of the breakwater are related to the state of breakwater foundation, figure 3.

Fixed supported bottom $\begin{cases} U(z = -d) = 0 \\ V(z = -d) = 0 \\ \gamma(z = -d) = 0 \end{cases}$ whatever the value of x. The problem arrives to 11 variables with 11 equations which describe the dynamical physical state of the wave-breakwater interaction. Solving for these variables, it is ended with the hydrodynamic stress tensor.

Hydrostatic pressure

The hydrostatic pressure exerted on the breakwater due to water depth has a linear form which simplifies the problem (figure 4). The difference between the water levels on each side of the breakwater is due to the elevation of the water level from the ocean side due to waves' propagations (h≈1.25 wave height). The stress tensor for the hydrostatic problem is supposed as follows:

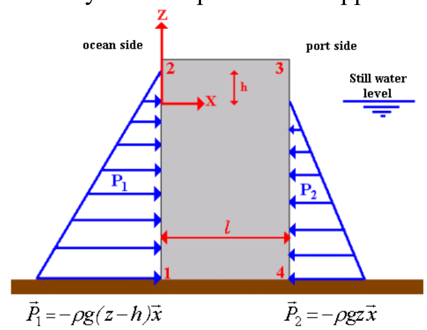


Figure 4 Hydrostatic pressure distribution over the breakwater

$$\sigma_{xs} = A_1 x^2 + B_1 z + C_1$$
$$\sigma_{zs} = A_2 xz(x - l) + B_2(z - h) + C_2 x(x - l) + D_2$$
$$\tau_{xzs} = A_3 xz(x - l) + B_3 x(z - h)^2 + C_3 x^2 + D_3$$

Repeating the same procedure for the hydrostatic pressure problem, 11 variables with 11 equations, it is ended by the hydrostatic stress tensor definition.

The global problem defining the real existing case is attained by combining the hydrostatic and hydrodynamic problem. This leads to an analytical addition for the resultant stresses defining the whole problem.

$$\sigma_x = \sigma_{xs} + \sigma_{xd} , \quad \sigma_z = \sigma_{zs} + \sigma_{zd} , \quad \tau_{xz} = \tau_{xzs} + \tau_{xzd}$$

The stress tensor is then finally defined by extremely complicated equations in function of the wave properties and the coordinates of any point on or inside the contour of the breakwater. In fact, it is very important to end up with such equations capable to define the stresses at any point of this breakwater, where it is deduced from analytical models. This constitutes an important base for the future work treating the optimisation of structure design and material of breakwaters. (where st , sm , ms are functions of x and z used in the σ_z expression to simplify it's writing and do not hold any physical explanation):

$$sm = \frac{(x - l)^4}{4(d + h)^3 \left[\dfrac{\cosh k(x - l)}{k} + \dfrac{xk^2 d(d + h) \sinh k(x - l)}{4} \right]} + \frac{4(x - l)^3}{k^2 h(d + h)^4 \sinh k(x - l)}$$

$$ms = \frac{k^2 (d + h)^4 \sinh k(x - l)}{4} \left[\cosh k(z + d) + kz \sinh k(z + d) + 3(z - h)^2 \sinh k(x - l) \right]$$

$$st = \frac{\cosh k(x - l)}{k} + \frac{xk^2 d(d + h) \sinh k(x - l)}{4}$$

$$\sigma_x = \frac{[achk(z + d) + bch2k(z + d) + f]}{shkl} shk(x - l) + \rho gh \frac{x^2}{l^2} + \rho gz - \rho gh$$

$$
\sigma_z = \left\{ \begin{array}{l}
\left[-\frac{x^4}{4sm(d+h)^3 st} - \frac{4x^3\cosh 2k(h+d)}{hk^2 sm(d+h)^4\cosh k(h+d)\sinh k(x-l)} \right] \times \\
\left[\begin{array}{l} (x-l)^3\cosh k(z+d) + \frac{(x-l)^4}{4(d+h)^3 st}[(z-h)^3\sinh k(x-l) \\ + k^2(d+h)^4 \sinh k(x-l) z\cosh k(z+d)] \end{array}\right] \\
+ x^3\cosh 2k(z+d) + \frac{x^4}{4(d+h)^3 st}\left[\begin{array}{l}(z-h)^3\sinh k(x-l)+\\ k^2(d+h)^4\sinh k(x-l)z\cosh k(z+d)\end{array}\right]
\end{array}\right\}
$$

$$
\times \frac{\left\{\begin{array}{l} \rho_b g - kz^2 \sinh k(4fdk2x-l) - 2x\cosh 2k(z+d) - 2(x-l)\cosh k(z+d) + \\ \frac{ms\upsilon fk^2d^2x\sinh k(x-l)}{(d+h)^3\sinh(kl)st} \\ + \frac{ms(a+b+f)\cosh k(x-l)}{(d+h)^3 k\upsilon\sinh(kl)st} - \frac{\upsilon fdk^2\sinh k(x-l)[\cosh k(z+d)+kz\sinh k(z+d)]}{\sinh(kl)} \\ -\left[\frac{ms(x-l)^4}{4(d+h)^3st} + k(x-l)^3\sinh k(z+d)\right]\times \\ \left[\frac{\upsilon fk^2d^2x\sinh k(x-l)}{sm(d+h)^3\sinh(kl)st} + \frac{(a+b+f)\cosh k(x-l)}{sm(d+h)^3k\upsilon\sinh(kl)st} - \frac{4\upsilon fdk^2}{smk^2(d+h)^4\sinh(kl)}\right]\end{array}\right\}}{\left\{\begin{array}{l} \frac{msx^4}{4(d+h)^3st} + 2kx^3\sinh 2k(z+d) - \left[\frac{ms(x-l)^4}{4(d+h)^3st}+k(x-l)^3\sinh k(z+d)\right]\times \\ \left[\begin{array}{l}\frac{x^4}{4sm(d+h)^3st}+\\ \frac{4x^3\cosh 2k(h+d)}{stk^2hsm(d+h)^4\cosh k(h+d)\sinh k(x-l)}\end{array}\right]\end{array}\right\}}
$$

$$
+\left[\begin{array}{l}(x-l)^3\cosh k(z+d) + \frac{(x-l)^4}{4(d+h)^3st}[(z-h)^3\sinh k(x-l)+ \\ k^2(d+h)^4\sinh k(x-l)z\cosh k(z+d)]\end{array}\right]\times
$$

$$
\left[\frac{(a+b+f)\cosh k(x-l)}{sm(d+h)^3k\upsilon\sinh(kl)st} + \frac{\upsilon fk^2d^2x\sinh k(x-l)}{st(d+h)^3\sinh(kl)sm} - \frac{4\upsilon fd}{sm(d+h)^4\sinh(kl)}\right]+
$$

$$
[(z-h)^3\sinh k(x-l) + k^2(d+h)^4\sinh k(x-l)z\cosh k(z+d)]\times
$$

$$
\left[-\frac{\upsilon fk^2d^2x\sinh k(x-l)}{st(d+h)^3\sinh(kl)} - \frac{(a+b+f)\cosh k(x-l)}{(d+h)^3k\upsilon\sinh(kl)st} + \frac{\upsilon fdk^2\sinh k(x-l)z\cosh k(z+d)}{\sinh(kl)}\right]
$$

$$
+\left\{\frac{\rho_b g + \frac{\rho gh(z-h)}{l}\left[1-\frac{2x}{l}\right] + \frac{2\upsilon\rho ghx^2}{l^2(2h+d)} - \frac{\upsilon\rho gd}{(2h+d)} - \frac{2\upsilon\rho gh}{(2h+d)}}{\left[x(x-l)\left(1-\frac{2h+2d}{2h+d}\right)\right]}\right\}\times
$$

$$
x(x-l)(z-h)\left(1-\frac{2h+2d}{2h+d}\right) - \frac{2\upsilon\rho ghx^2(z-h)}{l^2(2h+d)} + \frac{\upsilon\rho gd(z-h)}{2h+d} + \frac{2\upsilon\rho gh(z-h)}{2h+d}
$$

$$
\tau_{xz} = \frac{\begin{array}{l}\left[-k\cosh k(x-l)\frac{[a\cosh k(z+d)+b\cosh 2k(z+d)+f]}{\sinh kl}\right]\times \\ \left[z^2\left(\cosh k(x-l) - \frac{x^2}{l^2} - \frac{(x-l)^2\cosh kl}{l^2}\right) + z\cosh k(z+d)\left(1-\frac{x^2}{l^2}-\frac{(x-l)^2}{l^2}\right)\right]\end{array}}{\left\{\begin{array}{l}\left[\frac{-h^2\cosh k(x-l) + \frac{x^2z^2\cosh 2k(h+d)}{l^2\cosh 2k(z+d)} + \frac{z^2(x-l)^2\cosh k(h+d)\cosh kl}{l^2\cosh k(z+d)}}{h\cosh k(h+d) - \frac{zx^2\cosh k(z+d)\cosh 2k(h+d)}{l^2\cosh 2k(z+d)} - \frac{z(x-l)^2\cosh k(z+d)\cosh k(h+d)}{l^2\cosh k(z+d)}}\right]\times \\ \left[\cosh k(z+d) + kz\sinh k(z+d) - \frac{2kx^2\sinh 2k(z+d)}{l^2\cosh 2k(z+d)} - \frac{k(x-l)^2\sinh k(z+d)}{l^2\cosh k(z+d)}\right] \\ + 2z\cosh k(x-l) - \frac{2kx^2\sinh 2k(z+d)z^2}{l^2\cosh 2k(z+d)} - \frac{k(x-l)^2\sinh k(z+d)z^2\cosh kl}{l^2\cosh k(z+d)}\end{array}\right\}}
$$

$$
-\frac{2\rho gh(z-h)x}{l} + \frac{2gh(z-h)x^2}{l^2}
$$

2.3 Stability against overturning

After defining all the exerted forces on the breakwater and the stresses' distributions inside it, it is important to introduce a stability condition to maintain equilibrium despite all these exterior forces. The verification equation for structure stability, is the well known condition for overturning around the lower inner edge of the breakwater. $M_w - M_d - M_s \geq 0$; where M_w is the moment of the weight, M_d is the moment of the hydrodynamic pressure applied by the waves, and M_s is the moment of the hydrostatic pressure acting on the whole structure $(P_1 - P_2)$. From this stability condition, the width of the vertical breakwater can be simply deduced to stabilize it against overturning from the following expression: (ρ_m designates the density of the inside material)

$$
l^2 \geq \frac{2}{\rho_m(d+h)}\int_0^{d+h}(a\cosh k(z+d) + b\cosh 2k(z+d) + f)zdz
$$

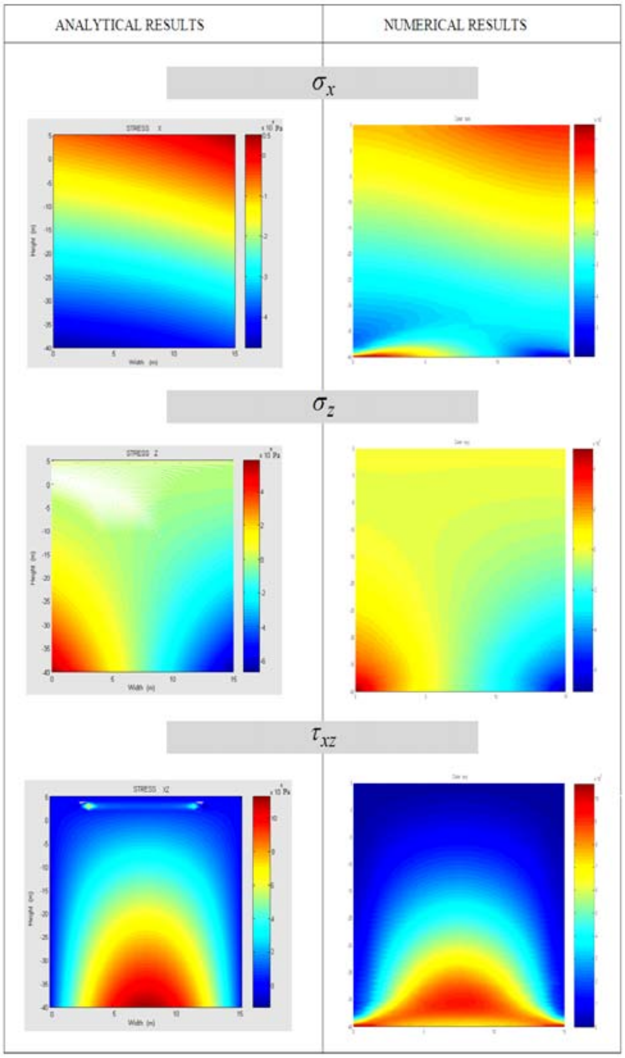
$$
+\frac{2}{\rho_m(d+h)}\left[\int_d^{d+h}(\rho(-z+d+h)zdz + \int_0^d[(\rho h)zdz\right]
$$

3-Verification of results

The success of the theoretical and analytical formulation of the wave-vertical breakwater interaction relies on the accuracy and accordance of these results with the numerical ones. The setup for an analytical calculation is as flows (choosing the parameters of a strong wave): wave properties [L=140 m, T=9 sec, d=40 m, H=4 m, r=0.8, t=0, sea water density=1025 kg/m^3] and breakwater properties [l=15 m (satisfying stability condition), concrete density=2300 kg/m^3, υ=0.2, E= 20x10^9 N/m^2], where all the stress equations are programmed in MATLAB inorder to benefit from its high quality in treating symbolic equations and drawing their evaluated contours. To verify our results, a comparison made with numerical results based on the finite element theory also using MATLAB in its partial differential equation toolbox.

4-Conclusion

An analytical model based on the potential flow theory for the wave propagation and on the mechanics of continuous medium for the stress distribution inside the structure, has been set up to study the wave-vertical breakwater interaction in this paper. By comparing with numerical data (pdetool MATLAB), this analytical model has been proven to be reliable to simulate the waves propagations, their induced pressures on the breakwater, and the structure behaviour due to the applied forces. Regardless to the difference in the stress distribution at the breakwater fixed support, the results of the analytical model are in good agreement with the numerical data.



Finally, there are some important remarks to be concluded from these interesting results and some recommendations for future work:

1- The maximum stresses' values result from the huge weight of the structure, from the hydrostatic pressure due to the water depth, and not from the hydrodynamic pressure induced form the waves' propagations, and hence the breakwater that is mainly designed to protect the ports from the waves' propagations is submitting induced pressure out of its objective that is playing an important role in characterising its design and overwhelming over all other constraints. For example, the maximum hydrodynamic pressure obtained from the above wave properties is 0.6 Pa where the hydrostatic one reaches 4.5 Pa at the bottom of the breakwater (see σ_x) and this results correspondingly in very high bending and shearing stresses reaching (see σ_z and τ_{xz}). These high values for bending stresses can probably cause the total destruction of the whole breakwater in case of strong waves due to the great traction efforts 4 MPa at the left bottom.

2-It is clearly observed that the difference in the stress contours distribution, near the fixed support between the two cases, returns to the two different methods in calculating the stress; where the support conditions appear only in the stress σ_z for the case of analytical modelling. On the other side, the numerical modelling takes into considerations the support conditions in calculating all the stresses by the method of finite element.

3- The future work must be oriented towards the floating breakwater instead of the fixed one, to suppress the great height of the breakwater causing the enormous hydrostatic pressure and the enormous weight of the structure; moreover to suppress the second important mode of failure summarised in the erosion of the seabed in the vicinity of the structure.

4- Determining the stress tensor at any point of the breakwater constitutes an important layout to analyse the load distribution inside it, and to move forward toward redesigning and optimising its external geometrical shape benefiting from the non uniform load distribution.

5- The computation must be approached using the finite element theory to attain the maximum precision in the studied model.

6- Choosing another material type for the breakwater lighter than concrete.

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